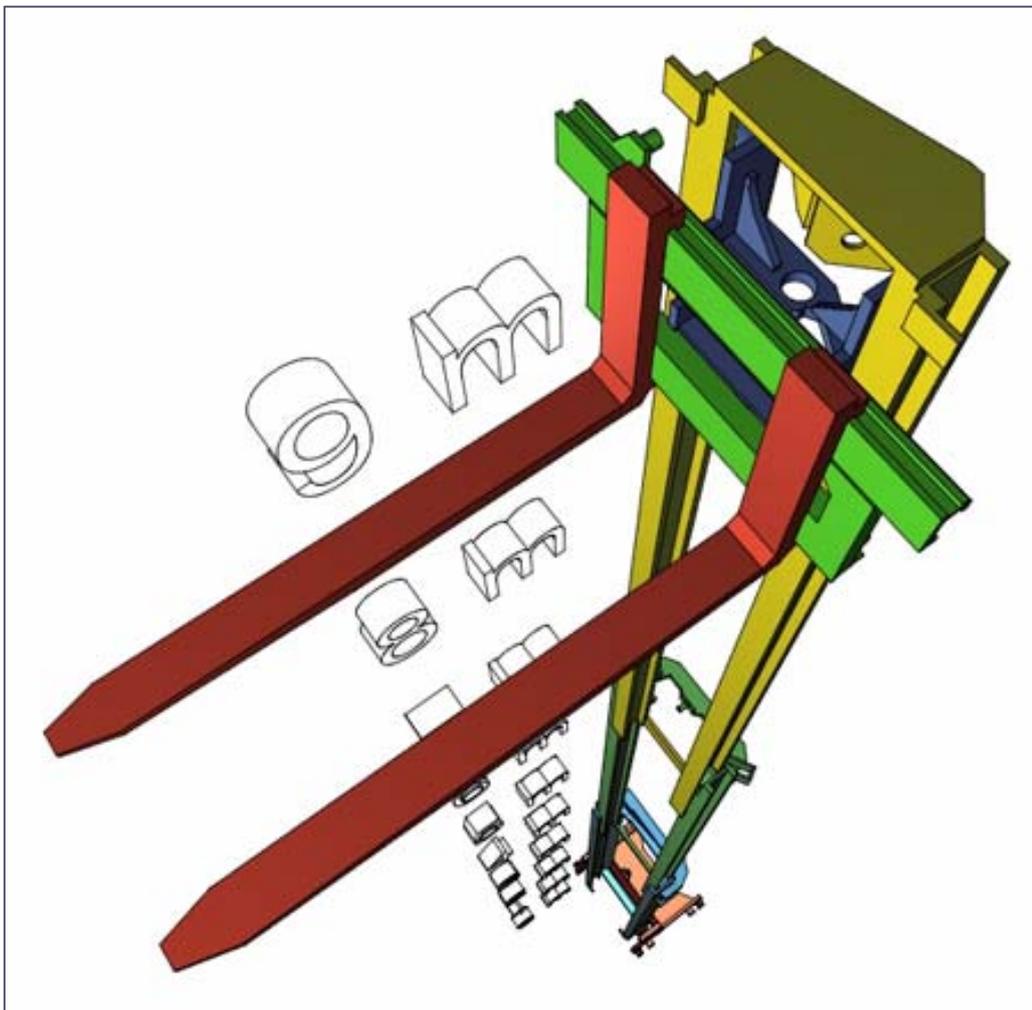


# TUTORIAL

## Introduction to Object-Oriented Modeling and Simulation with OpenModelica

Peter Fritzson



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Version 2006



## Abstract

Object-Oriented modeling is a fast-growing area of modeling and simulation that provides a structured, computer-supported way of doing mathematical and equation-based modeling. Modelica is today the most promising modeling and simulation language in that it effectively unifies and generalizes previous object-oriented modeling languages and provides a sound basis for the basic concepts.

The Modelica modeling language and technology is being warmly received by the world community in modeling and simulation with major applications in virtual prototyping. It is bringing about a revolution in this area, based on its ease of use, visual design of models with combination of lego-like predefined model building blocks, its ability to define model libraries with reusable components, its support for modeling and simulation of complex applications involving parts from several application domains, and many more useful facilities. To draw an analogy, Modelica is currently in a similar phase as Java early on, before the language became well known, but for virtual prototyping instead of Internet programming.

The tutorial presents an object-oriented component-based approach to computer supported mathematical modeling and simulation through the powerful Modelica language and its associated technology. Modelica can be viewed as an almost universal approach to high level computational modeling and simulation, by being able to represent a range of application areas and providing general notation as well as powerful abstractions and efficient implementations.

The tutorial gives an introduction to the Modelica language to people who are familiar with basic programming concepts. It gives a basic introduction to the concepts of modeling and simulation, as well as the basics of object-oriented component-based modeling for the novice, and an overview of modeling and simulation in a number of application areas.

The tutorial has several goals:

- Being easily accessible for people who do not previously have a background in modeling, simulation.
- Introducing the concepts of physical modeling, object-oriented modeling and component-based modeling and simulation.
- Giving an introduction to the Modelica language.
- Demonstrating modeling examples from several application areas.
- Giving a possibility for hands-on exercises.

## Presenter's data

**Peter Fritzson** is a Professor and Director of the Programming Environment Laboratory (Pelab), at the Department of Computer and Information Science, Linköping University, Sweden. He holds the position of Director of Research and Development of MathCore Engineering AB. Peter Fritzson is chairman of the Scandinavian Simulation Society, secretary of the European simulation organization, EuroSim; and vice chairman of the Modelica Association, an organization he helped to establish. His main area of interest is software engineering, especially design, programming and maintenance tools and environments.

## 1. Useful Web Links

The Modelica Association Web Page

<http://www.modelica.org>

Modelica publications

<http://www.modelica.org/publications.shtml>

Modelica related research and the OpenModelica open source project at Linköping University with download of the OpenModelica system and link to download of MathModelica Lite.

<http://www.ida.liu.se/~pelab/modelica/OpenModelica.html>

The Proceedings of 5th International Modelica Conference, September 4-5, 2006, Vienna, Austria

<http://www.modelica.org/events/Conference2006/>

The Proceedings of 4th International Modelica Conference, March 7-8, 2005, Hamburg, Germany

<http://www.modelica.org/events/Conference2005/>

The Proceedings of 3rd International Modelica Conference, November 3-4, 2004, Linköping, Sweden

<http://www.modelica.org/events/Conference2003/>

The Proceedings of 2nd International Modelica Conference, March 18-19, 2002, "Deutsches Zentrum für Luft- und Raumfahrt" at Oberpfaffenhofen, Germany.

<http://www.modelica.org/events/Conference2002/>

The Proceedings of Modelica Workshop, October 23 - 24, 2000, Lund University, Lund, Sweden

<http://www.modelica.org/events/workshop2000/>

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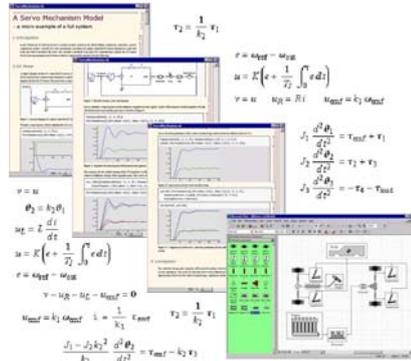
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# Principles of Object-Oriented Modeling and Simulation with Modelica



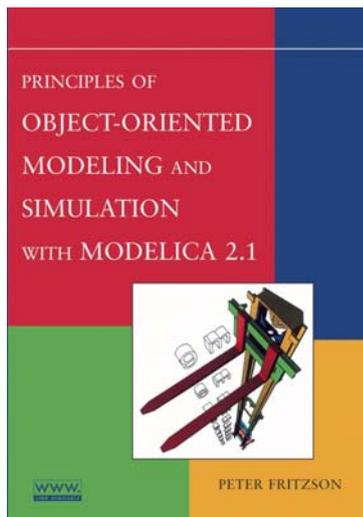
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pelab

MODELICA

## Course Based on Recent Book, 2004



**Peter Fritzson**  
**Principles of Object Oriented Modeling and Simulation with Modelica 2.1**

Wiley-IEEE Press

940 pages

## Acknowledgements, Usage, Copyrights

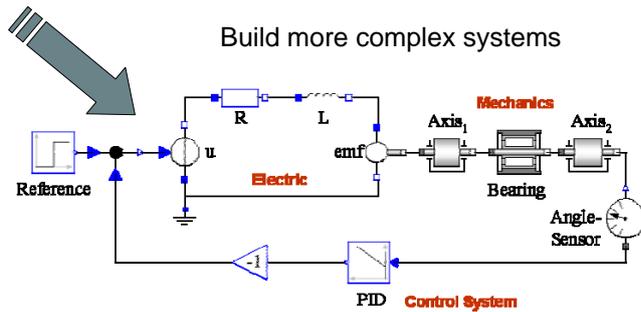
- If you want to use the Powerpoint version of these slides in your own course, send an email to:  
[peter.fritzson@ida.liu.se](mailto:peter.fritzson@ida.liu.se)
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- OpenModelica: [www.ida.liu.se/projects/OpenModelica](http://www.ida.liu.se/projects/OpenModelica)

## Outline

- Introduction to Modeling and Simulation
- Modelica - The next generation modeling and Simulation Language
- Classes
- Components, Connectors and Connections
- Equations
- Discrete Events and Hybrid Systems
- Algorithm and Functions
- Modeling and Simulation Environments
- Demonstrations

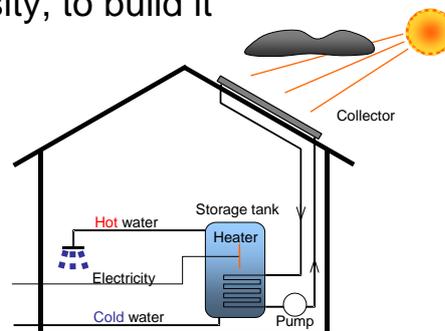
## Why Modeling & Simulation ?

- Increase understanding of complex systems
- Design and optimization
- Virtual prototyping
- Verification



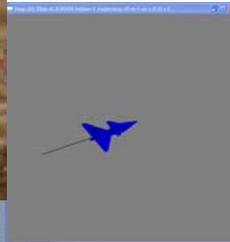
## What is a system?

- A system is an object or collection of objects whose properties we want to study
- Natural and artificial systems
- Reasons to study: curiosity, to build it



## Examples of Complex Systems

- Robotics
- Automotive
- Aircrafts
- Satellites
- Biomechanics
- Power plants
- Hardware-in-the-loop, real-time simulation



## Experiments

An *experiment* is the process of extracting information from a system by exercising its inputs

### Problems

- Experiment might be too *expensive*
- Experiment might be too *dangerous*
- System needed for the experiment might *not yet exist*

## Model concept

A *model* of a system is anything an *experiment* can be applied to in order to answer questions about that *system*

Kinds of models:

- **Mental model** – statement like “a person is reliable”
- **Verbal model** – model expressed in words
- **Physical model** – a physical object that mimics the system
- **Mathematical model** – a description of a system where the relationships are expressed in mathematical form – a *virtual prototype*
- **Physical modeling** – also used for mathematical models built/structured in the same way as physical models

## Simulation

A *simulation* is an *experiment* performed on a *model*

Examples of simulations:

- **Industrial process** – such as steel or pulp manufacturing, study the behaviour under different operating conditions in order to improve the process
- **Vehicle behaviour** – e.g. of a car or an airplane, for operator training
- **Packet switched computer network** – study behaviour under different loads to improve performance

## Reasons for Simulation

- Suppression of *second-order effects*
- Experiments are too *expensive*, too *dangerous*, or the system to be investigated does *not yet exist*
- The *time scale* is not compatible with experimenter (Universe, million years, ...)
- Variables may be *inaccessible*.
- Easy *manipulation* of models
- Suppression of *disturbances*

## Dangers of Simulation

### Falling in love with a model

The Pygmalion effect (forgetting that model is not the real world, e.g. introduction of foxes to hunt rabbits in Australia)

### Forcing reality into the constraints of a model

The Procrustes effect (e.g. economic theories)

### Forgetting the model's level of accuracy

Simplifying assumptions

## Building Models Based on Knowledge

### System knowledge

- The collected *general experience* in relevant domains
- The *system* itself

### Specific or generic knowledge

- E.g. software engineering knowledge

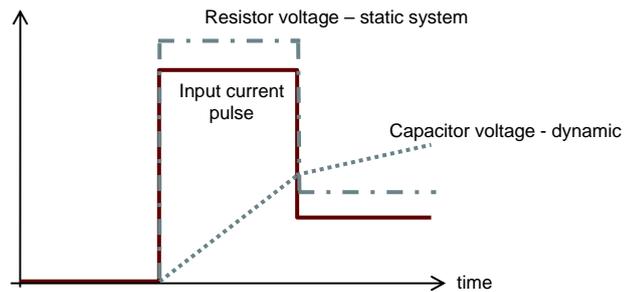
## Kinds of Mathematical Models

- Dynamic vs. Static models
- Continuous-time vs. Discrete-time dynamic models
- Quantitative vs. Qualitative models

## Dynamic vs. Static Models

A **dynamic** model includes *time* in the model

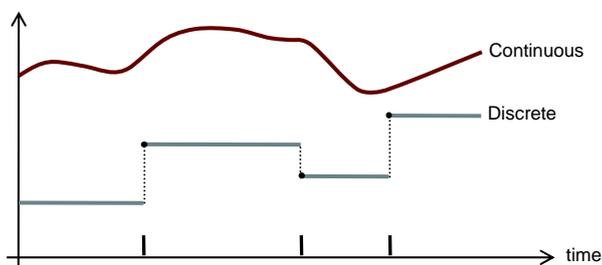
A **static** model can be defined *without* involving *time*



## Continuous-Time vs. Discrete-Time Dynamic Models

**Continuous-time** models may evolve their variable values *continuously* during a time period

**Discrete-time** variables change values a *finite* number of times during a time period

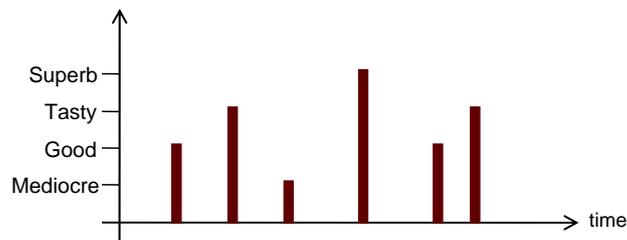


## Quantitative vs. Qualitative Models

### Results in qualitative data

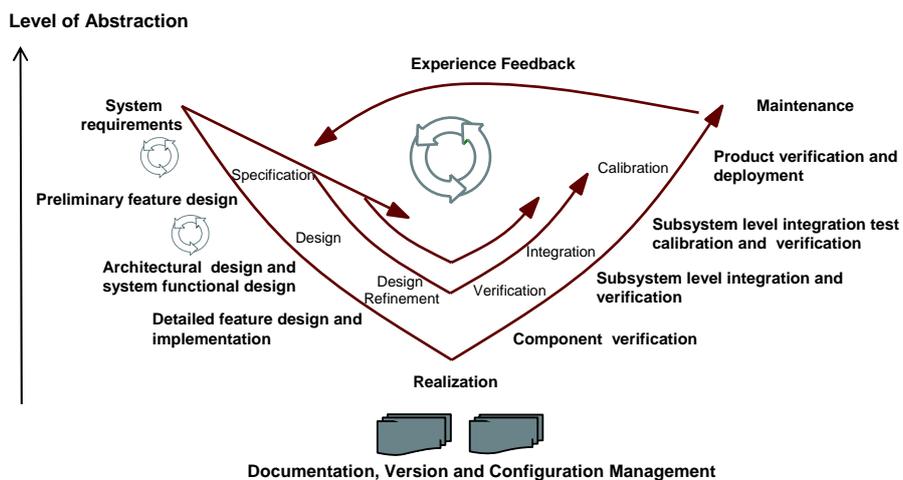
Variable values cannot be represented numerically

Mediocre = 1, Good = 2, Tasty = 3, Superb = 4



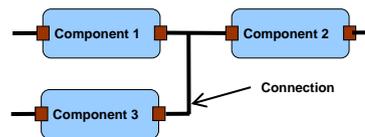
Quality of food in a restaurant according to inspections at irregular points in time

## Using Modeling and Simulation within the Product Design-V

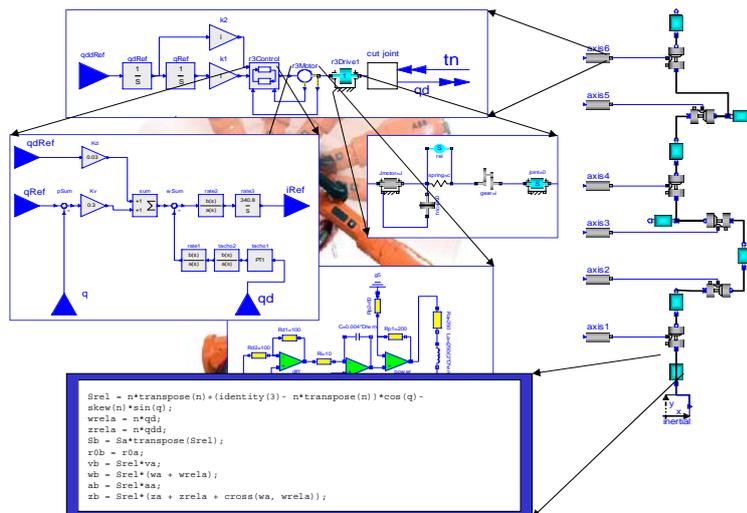


## Principles of Equation-Based Modeling

- Each icon represents a physical component  
i.e. Resistor, mechanical Gear Box, Pump
- Composition lines represent the actual physical connections i.e. electrical line, mechanical connection, heat flow
- Variables at the interfaces describe interaction with other component
- Physical behavior of a component is described by equations
- Hierarchical decomposition of components

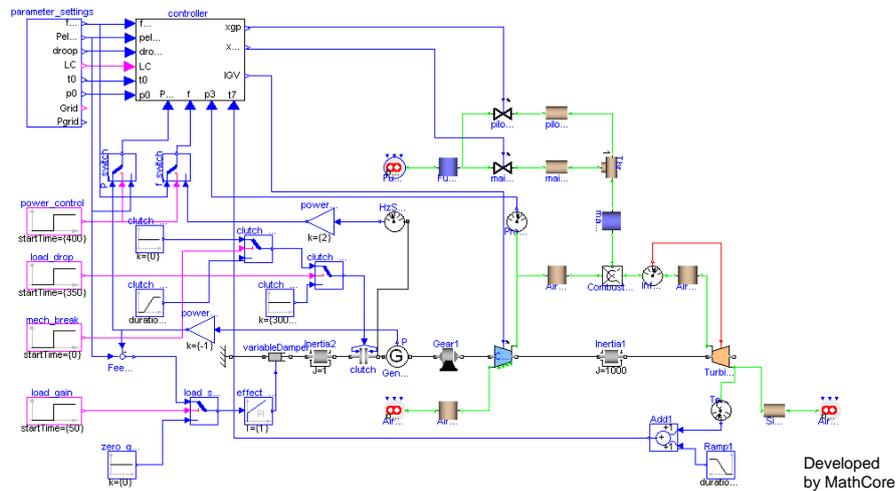


## Application Example – Industry Robot



Courtesy of Martin Otter

## GTX Gas Turbine Power Cutoff Mechanism



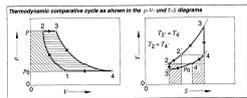
Courtesy of Siemens Industrial Turbomachinery AB

## Modelica – The Next Generation Modeling Language

## Stored Knowledge

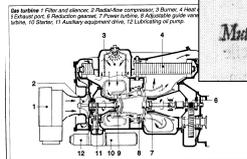
Model knowledge is stored in books and human minds which computers cannot access

Internal-combustion engines 417



Let  $T_1$  to  $T_2$ , supplied by the heat exchanger is coupled with a thermal reservoir  $T_1 = T_2$ . If heat is completely exchanged, the quantity of heat to be absorbed  $Q_1$  is reduced to  $Q_1 = Q_2 (T_1 - T_2) / (T_1 - T_2)$  and the quantity of heat to be rejected is  $Q_2 = Q_1 (T_1 - T_2) / (T_1 - T_2)$ . The maximum thermal efficiency for the cycle with heat exchanger is  $\eta_{th} = 1 - (T_2 - T_1) / (T_1 - T_2)$ .

*“The change of motion is proportional to the motive force impressed”*  
– Newton



Lex. II.

*Mutationem motus proportionalem esse vi motrici impressae, & fieri secundum lineam rectam qua vis illa imprimatur.*

## The Form – Equations

- Equations were used in the third millennium B.C.
- Equality sign was introduced by Robert Recorde in 1557

14.ze. — + — . 15.9 — = = = 71.9.

Newton still wrote text (Principia, vol. 1, 1686)

*“The change of motion is proportional to the motive force impressed”*

CSSL (1967) introduced a special form of “equation”:

variable = expression

$v = \text{INTEG}(F) / m$

**Programming languages usually do not allow equations!**

## Modelica – The Next Generation Modeling Language

### Declarative language

Equations and mathematical functions allow acausal modeling, high level specification, increased correctness

### Multi-domain modeling

Combine electrical, mechanical, thermodynamic, hydraulic, biological, control, event, real-time, etc...

### Everything is a class

Strongly typed object-oriented language with a general class concept, Java & MATLAB-like syntax

### Visual component programming

Hierarchical system architecture capabilities

### Efficient, non-proprietary

Efficiency comparable to C; advanced equation compilation, e.g. 300 000 equations, ~150 000 lines on standard PC

## Modelica – The Next Generation Modeling Language

### High level language

MATLAB-style array operations; Functional style; iterators, constructors, object orientation, equations, etc.

### MATLAB similarities

MATLAB-like array and scalar arithmetic, but strongly typed and efficiency comparable to C.

### Non-Proprietary

- Open Language Standard
- Both Open-Source and Commercial implementations

### Flexible and powerful external function facility

- LAPACK interface effort started

## Modelica Language Properties

- **Declarative** and **Object-Oriented**
- **Equation-based**; continuous and discrete equations
- **Parallel** process modeling of real-time applications, according to synchronous data flow principle
- **Functions** with algorithms without global side-effects (but local data updates allowed)
- **Type system** inspired by Abadi/Cardelli
- **Everything is a class** – Real, Integer, models, functions, packages, parameterized classes....

## Object Oriented Mathematical Modeling with Modelica

- The static *declarative structure* of a mathematical model is emphasized
- OO is primarily used as a *structuring concept*
- OO *is not* viewed as dynamic object creation and sending messages
- *Dynamic model* properties are expressed in a *declarative way* through equations.
- Acausal classes supports *better reuse of modeling and design knowledge* than traditional classes

## Brief Modelica History

- First Modelica design group meeting in fall 1996
  - International group of people with expert knowledge in both language design and physical modeling
  - Industry and academia
- Modelica Versions
  - 1.0 released September 1997
  - 2.0 released March 2002
  - Latest version, 2.2 released March 2005
- Modelica Association established 2000
  - Open, non-profit organization

## Modelica Conferences

- The 1<sup>st</sup> International Modelica conference October, 2000
- The 2<sup>nd</sup> International Modelica conference March 18-19, 2002
- The 3<sup>rd</sup> International Modelica conference November 5-6, 2003 in Linköping, Sweden
- The 4<sup>th</sup> International Modelica conference March 6-7, 2005 in Hamburg, Germany
- The 5<sup>th</sup> International Modelica conference planned September 4-5, 2006 in Vienna, Austria

# Modelica Classes and Inheritance

## Simplest Model – Hello World!

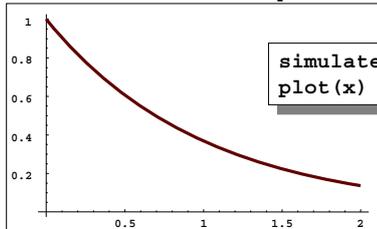
### A Modelica “Hello World” model

Equation:  $x' = -x$

Initial condition:  $x(0) = 1$

```
class HelloWorld "A simple equation"  
  Real x(start=1);  
equation  
  der(x) = -x;  
end HelloWorld;
```

### Simulation in OpenModelica environment



```
simulate(HelloWorld, stopTime = 2)  
plot(x)
```

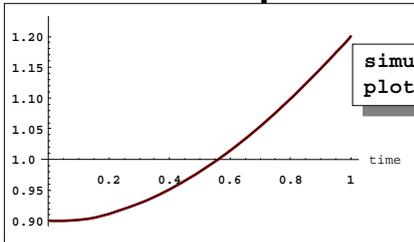
## Another Example

### Include algebraic equation

Algebraic equations contain  
no derivatives

```
class DAEexample
  Real x(start=0.9);
  Real y;
  equation
    der(y)+(1+0.5*sin(y))*der(x)
      = sin(time);
    x - y = exp(-0.9*x)*cos(y);
end DAEexample;
```

### Simulation in OpenModelica environment

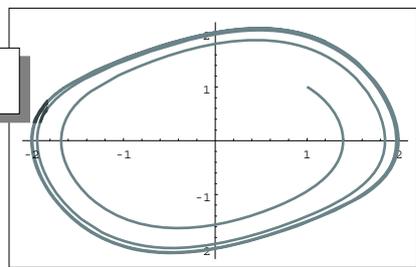


```
simulate(DAEexample, stopTime = 1)
plot(x)
```

## Example class: Van der Pol Oscillator

```
class VanDerPol "Van der Pol oscillator model"
  Real x(start = 1) "Descriptive string for x"; // x starts at 1
  Real y(start = 1) "y coordinate"; // y starts at 1
  parameter Real lambda = 0.3;
  equation
    der(x) = y; // This is the 1st diff equation //
    der(y) = -x + lambda*(1 - x*x)*y; /* This is the 2nd diff equation */
end VanDerPol;
```

```
simulate(VanDerPol, stopTime = 25)
plotParametric(x, y)
```



## Small Exercise

- Locate the HelloWorld model in DrModelica using OMNotebook!
- Simulate and plot the example. Do a slight change in the model, re-simulate and re-plot.

```
class HelloWorld "A simple equation"  
  Real x(start=1);  
equation  
  der(x) = -x;  
end HelloWorld;
```

```
simulate(HelloWorld, stopTime = 2)  
plot(x)
```

- Locate the VanDerPol model in DrModelica and try it!

## Variables and Constants

### Built-in primitive data types

<b>Boolean</b>	true or false
<b>Integer</b>	Integer value, e.g. 42 or -3
<b>Real</b>	Floating point value, e.g. 2.4e-6
<b>String</b>	String, e.g. "Hello world"
<b>Enumeration</b>	Enumeration literal e.g. <b>ShirtSize.Medium</b>

## Variables and Constants cont'

- Names indicate meaning of constant
- Easier to maintain code
- Parameters are constant during simulation
- Two types of constants in Modelica
  - `constant`
  - `parameter`

```
constant Real    PI=3.141592653589793;  
constant String  redcolor = "red";  
constant Integer one = 1;  
parameter Real   mass = 22.5;
```

## Comments in Modelica

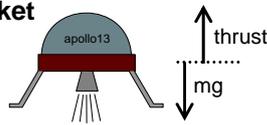
- 1) Declaration comments, e.g. `Real x "state variable";`

```
class VanDerPol "Van der Pol oscillator model"  
  Real x(start = 1) "Descriptive string for x"; // x starts at 1  
  Real y(start = 1) "y coordinate";           // y starts at 1  
  parameter Real lambda = 0.3;  
equation  
  der(x) = y; // This is the 1st diff equation //  
  der(y) = -x + lambda*(1 - x*x)*y; /* This is the 2nd diff equation */  
end VanDerPol;
```

- 2) Source code comments, disregarded by compiler
  - 2a) C style, e.g. `/* This is a C style comment */`
  - 2b) C++ style, e.g. `// Comment to the end of the line..`

## A Simple Rocket Model

Rocket



$$\text{acceleration} = \frac{\text{thrust} - \text{mass} \cdot \text{gravity}}{\text{mass}}$$

$$\text{mass}' = -\text{massLossRate} \cdot \text{abs}(\text{thrust})$$

$$\text{altitude}' = \text{velocity}$$

$$\text{velocity}' = \text{acceleration}$$

<p>new model ←</p> <p>parameters (changeable before the simulation) ←</p> <p>floating point type ←</p> <p>differentiation with regards to time ←</p>	<pre>class Rocket, "rocket_class"   parameter String name;   Real mass(start=1038.358);   Real altitude(start= 59404);   Real velocity(start= 2003);   Real acceleration;   Real thrust; // Thrust force on rocket   Real gravity; // Gravity forcefield   parameter Real massLossRate=0.000277; equation   ((thrust-mass*gravity)/mass==acceleration);   der(mass) = -massLossRate * abs(thrust);   der(altitude) = velocity;   der(velocity) = acceleration; end Rocket;</pre>	<p>→ declaration comment</p> <p>→ start value</p> <p>→ name + default value</p> <p>→ mathematical equation (acausal)</p>
--	--	--

## Celestial Body Class

A class declaration creates a *type name* in Modelica

```
class CelestialBody
  constant Real g = 6.672e-11;
  parameter Real radius;
  parameter String name;
  parameter Real mass;
end CelestialBody;
```

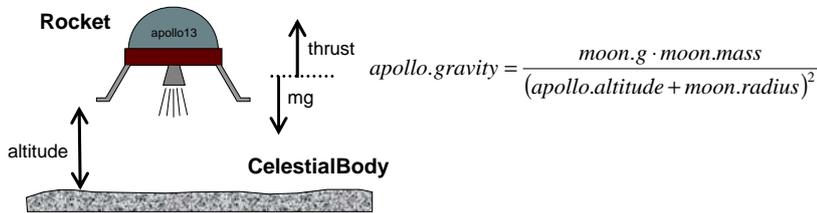


An *instance* of the class can be declared by *prefixing* the type name to a variable name

```
...
CelestialBody moon;
...
```

The declaration states that **moon** is a variable containing an object of type **CelestialBody**

## Moon Landing



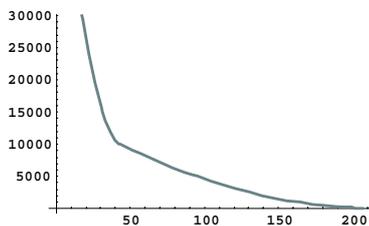
only access  
inside the class

access by dot  
notation outside  
the class

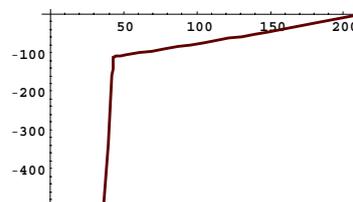
```
class MoonLanding
  parameter Real force1 = 36350;
  parameter Real force2 = 1308;
  protected
  parameter Real thrustEndTime = 210;
  parameter Real thrustDecreaseTime = 43.2;
  public
  Rocket
  CelestialBody
  apollo (name="apollo13");
  moon (name="moon", mass=7.382e22, radius=1.738e6);
  equation
  apollo.thrust = if (time < thrustDecreaseTime) then force1
    else if (time < thrustEndTime) then force2
    else 0;
  apollo.gravity=moon.g*moon.mass/(apollo.altitude+moon.radius)^2;
end MoonLanding;
```

## Simulation of Moon Landing

```
simulate(MoonLanding, stopTime=230)
plot(apollo.altitude, xrange={0,208})
plot(apollo.velocity, xrange={0,208})
```



It starts at an altitude of 59404 (not shown in the diagram) at time zero, gradually reducing it until touchdown at the lunar surface when the altitude is zero



The rocket initially has a high negative velocity when approaching the lunar surface. This is reduced to zero at touchdown, giving a smooth landing

## Restricted Class Keywords

- The `class` keyword can be replaced by other keywords, e.g.: `model`, `record`, `block`, `connector`, `function`, ...
- Classes declared with such keywords have restrictions
- Restrictions apply to the contents of restricted classes
- Example: A `model` is a class that cannot be used as a connector class
- Example: A `record` is a class that only contains data, with no equations
- Example: A `block` is a class with fixed input-output causality

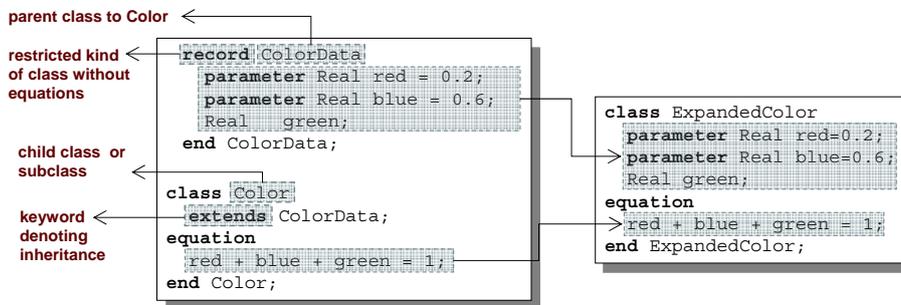
```
model CelestialBody
  constant Real    g = 6.672e-11;
  parameter Real   radius;
  parameter String name;
  parameter Real   mass;
end CelestialBody;
```

## Modelica Functions

- Modelica Functions can be viewed as a special kind of restricted class with some extensions
- A function can be called with arguments, and is instantiated dynamically when called
- More on functions and algorithms later in Lecture 4

```
function sum
  input Real arg1;
  input Real arg2;
  output Real result;
algorithm
  result := arg1+arg2;
end sum;
```

# Inheritance



Data and behavior: field declarations, equations, and certain other contents are copied into the subclass

# Inheriting definitions

```

record ColorData
  parameter Real red = 0.2;
  parameter Real blue = 0.6;
  Real green;
end ColorData;

class ErrorColor
  extends ColorData;
  >parameter Real blue = 0.6;
  >parameter Real red = 0.3;
  equation
    red + blue + green = 1;
end ErrorColor;

```

Legal!  
Identical to the inherited field blue

Inheriting multiple identical definitions results in only one definition

Illegal!  
Same name, but different value

Inheriting multiple different definitions of the same item is an error

## Inheritance of Equations

```
class Color
  parameter Real red=0.2;
  parameter Real blue=0.6;
  Real green;
  equation
    red + blue + green = 1;
end Color;
```

```
class Color2 // OK!
  extends Color;
  equation
    red + blue + green = 1;
end Color2;
```

```
class Color3 // Error!
  extends Color;
  equation
    red + blue + green = 1.0;
    // also inherited: red + blue + green = 1;
end Color3;
```

Color is identical to Color2  
 → Same equation twice leaves one copy when inheriting

Color3 is overdetermined  
 → Different equations means two equations!

## Multiple Inheritance

Multiple Inheritance is fine – inheriting both geometry and color

```
class Color
  parameter Real red=0.2;
  parameter Real blue=0.6;
  Real green;
  equation
    red + blue + green = 1;
end Color;
```

```
class Point
  Real x;
  Real y, z;
end Point;
```

```
class ColoredPoint
  extends Point;
  extends Color;
end ColoredPoint;
```

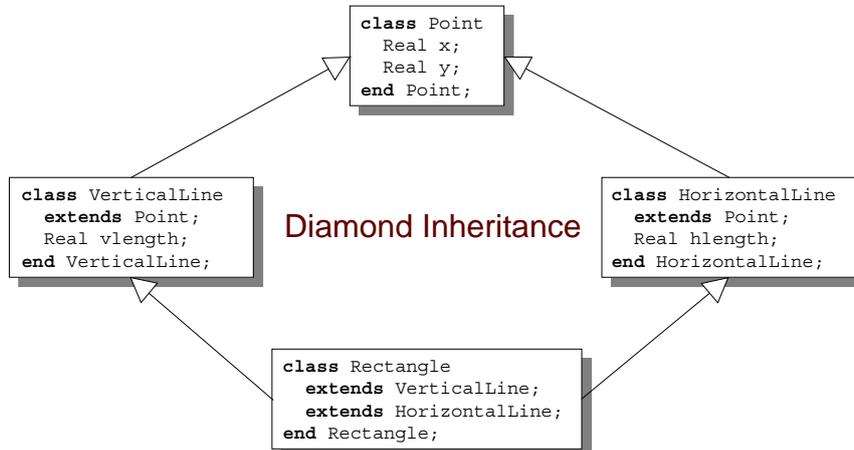
multiple inheritance

```
class ColoredPointWithoutInheritance
  Real x;
  Real y, z;
  parameter Real red = 0.2;
  parameter Real blue = 0.6;
  Real green;
  equation
    red + blue + green = 1;
end ColoredPointWithoutInheritance;
```

Equivalent to

## Multiple Inheritance cont'

Only one copy of multiply inherited class Point is kept



## Simple Class Definition – Shorthand Case of Inheritance

- Example:

```
class SameColor = Color;
```

Equivalent to:

```
class SameColor
  extends Color;
end SameColor;
```

inheritance ←

- Often used for introducing new names of types:

```
type Resistor = Real;
```

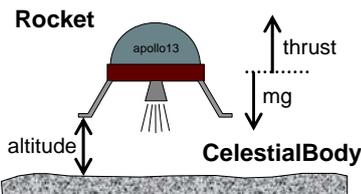
```
connector MyPin = Pin;
```

## Inheritance Through Modification

- Modification is a concise way of combining inheritance with declaration of classes or instances
- A modifier modifies a declaration equation in the inherited class
- Example: The class `Real` is inherited, modified with a different `start` value equation, and instantiated as an `altitude` variable:

```
...  
Real altitude(start= 59404);  
...
```

## The Moon Landing Example Using Inheritance



```
model Body "generic body"  
  Real mass;  
  String name;  
end Body;
```

```
model CelestialBody  
  extends Body;  
  constant Real g = 6.672e-11;  
  parameter Real radius;  
end CelestialBody;
```

```
model Rocket "generic rocket class"  
  extends Body;  
  parameter Real massLossRate=0.000277;  
  Real altitude(start= 59404);  
  Real velocity(start= -2003);  
  Real acceleration;  
  Real thrust;  
  Real gravity;  
equation  
  thrust-mass*gravity= mass*acceleration;  
  der(mass)= -massLossRate*abs(thrust);  
  der(altitude)= velocity;  
  der(velocity)= acceleration;  
end Rocket;
```

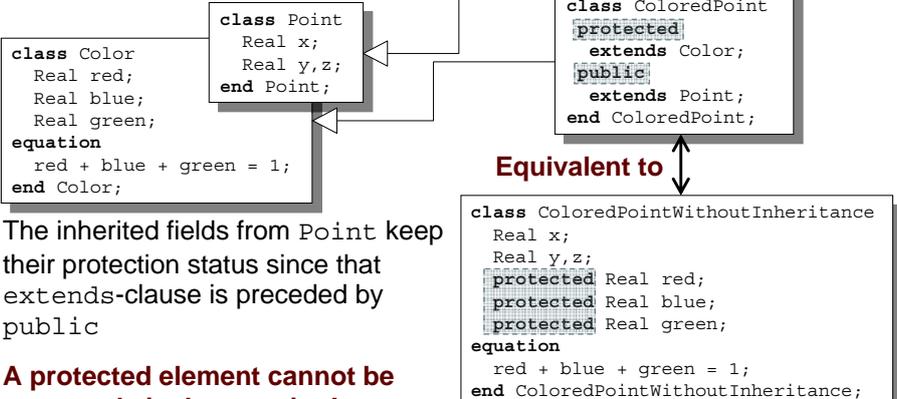
## The Moon Landing Example using Inheritance cont'

```

model MoonLanding
  parameter Real force1 = 36350;
  parameter Real force2 = 1308;
  parameter Real thrustEndTime = 210;
  parameter Real thrustDecreaseTime = 43.2;
  Rocket      apollo(name="apollo13", mass(start=1038.358) );
  CelestialBody moon(mass=7.382e22, radius=1.738e6, name="moon");
  equation
    apollo.thrust = if (time<thrustDecreaseTime) then force1
                    else if (time<thrustEndTime) then force2
                    else 0;
    apollo.gravity =moon.g*moon.mass/(apollo.altitude+moon.radius)^2;
  end Landing;
  
```

## Inheritance of Protected Elements

If an extends-clause is preceded by the `protected` keyword, all inherited elements from the superclass become protected elements of the subclass



## Advanced Topic

- Class parameterization

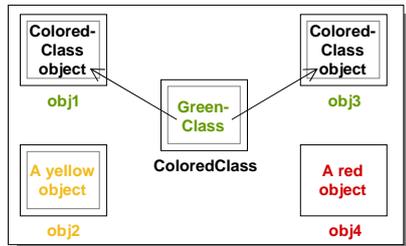
## Generic Classes with Type Parameters

Formal class parameters are replaceable variable or type declarations within the class (usually marked with the prefix `replaceable`)

Actual arguments to classes are modifiers, which when containing whole variable declarations or types are preceded by the prefix `redeclare`

```
class C
  replaceable class ColoredClass = GreenClass;
  ColoredClass obj1 (p1=5);
  replaceable YellowClass obj2;
  ColoredClass obj3;
  RedClass obj4;
equation
end C;
```

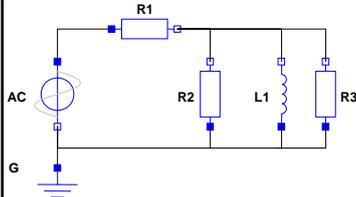
```
class C2 =
  C (redeclare class ColoredClass = BlueClass);
```



Equivalent to

```
class C2
  BlueClass obj1 (p1=5);
  YellowClass obj2;
  BlueClass obj3;
  RedClass obj4;
equation
end C2;
```

## Class Parameterization when Class Parameters are Components



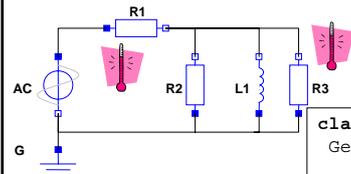
The class `ElectricalCircuit` has been converted into a parameterized generic class `GenericElectricalCircuit` with three formal class parameters `R1`, `R2`, `R3`, marked by the keyword `replaceable`

```
class ElectricalCircuit
  Resistor R1(R=100);
  Resistor R2(R=200);
  Resistor R3(R=300);
  Inductor L1;
  SineVoltage AC;
  Ground G;
equation
  connect(R1.n,R2.n);
  connect(R1.n,L1.n);
  connect(R1.n,R3.n);
  connect(R1.p,AC.p);
  .....
end ElectricalCircuit;
```

Class parameterization

```
class GenericElectricalCircuit
  replaceable Resistor R1(R=100);
  replaceable Resistor R2(R=200);
  replaceable Resistor R3(R=300);
  Inductor L1;
  SineVoltage AC;
  Ground G;
equation
  connect(R1.n,R2.n);
  connect(R1.n,L1.n);
  connect(R1.n,R3.n);
  connect(R1.p,AC.p);
  .....
end GenericElectricalCircuit;
```

## Class Parameterization when Class Parameters are Components - cont'



A more specialized class `TemperatureElectricalCircuit` is created by changing the types of `R1`, `R3`, to `TempResistor`

```
class TemperatureElectricalCircuit =
  GenericElectricalCircuit
  redeclare TempResistor R1;
  redeclare TempResistor R3;
```

```
class TemperatureElectricalCircuit
  parameter Real Temp=20;
  extends GenericElectricalCircuit(
    redeclare TempResistor R1(RT=0.1, Temp=Temp),
    redeclare TempResistor R3(R=300));
end TemperatureElectricalCircuit
```

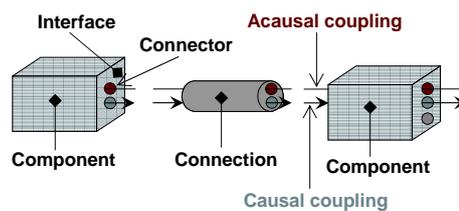
We add a temperature variable `Temp` for the temperature of the resistor circuit and modifiers for `R1` and `R3` which are now `TempResistors`.

equivalent to

```
class ExpandedTemperatureElectricalCircuit
  parameter Real Temp;
  TempResistor R1(R=200, RT=0.1, Temp=Temp),
  replaceable Resistor R2;
  TempResistor R3(R=300);
equation
  ....
end ExpandedTemperatureElectricalCircuit
```

# Components, Connectors and Connections

## Software Component Model



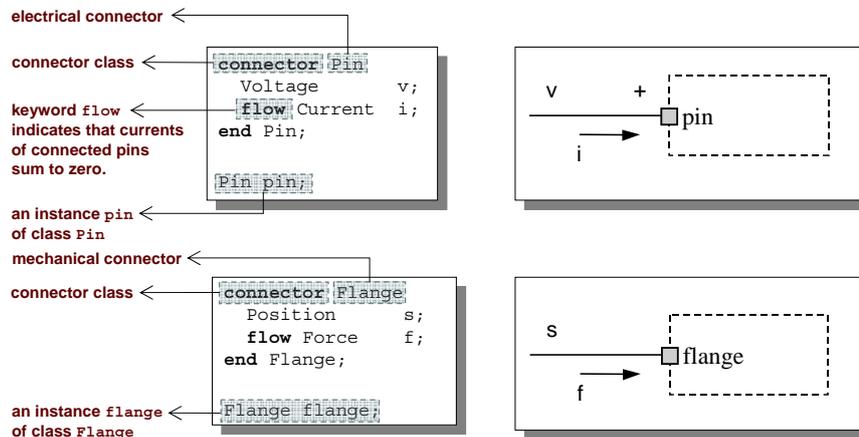
A component class should be defined *independently of the environment*, very essential for *reusability*

A component may internally consist of other components, i.e. *hierarchical* modeling

Complex systems usually consist of large numbers of *connected* components

## Connectors and Connector Classes

Connectors are instances of *connector classes*



## The flow prefix

Two kinds of variables in connectors:

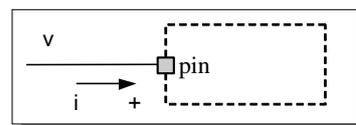
- *Non-flow variables* potential or energy level
- *Flow variables* represent some kind of flow

Coupling

- *Equality coupling*, for non-flow variables
- *Sum-to-zero coupling*, for flow variables

The value of a flow variable is *positive* when the current or the flow is *into* the component

positive flow direction:



## Physical Connector Classes Based on Energy Flow

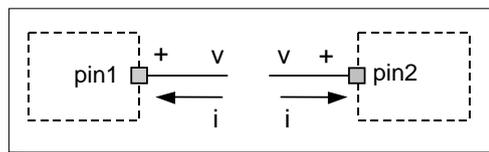
Domain Type	Potential	Flow	Carrier	Modelica Library
Electrical	Voltage	Current	Charge	Electrical. Analog
Translational	Position	Force	Linear momentum	Mechanical. Translational
Rotational	Angle	Torque	Angular momentum	Mechanical. Rotational
Magnetic	Magnetic potential	Magnetic flux rate	Magnetic flux	
Hydraulic	Pressure	Volume flow	Volume	HyLibLight
Heat	Temperature	Heat flow	Heat	HeatFlow1D
Chemical	Chemical potential	Particle flow	Particles	Under construction
Pneumatic	Pressure	Mass flow	Air	PneuLibLight

## connect-equations

Connections between connectors are realized as *equations* in Modelica

```
connect (connector1, connector2)
```

The two arguments of a `connect`-equation must be references to *connectors*, either to be declared directly *within* the *same class* or be *members* of one of the declared variables in that class



```
Pin pin1, pin2;  
//A connect equation  
//in Modelica:  
connect (pin1, pin2);
```

Corresponds to

```
pin1.v = pin2.v;  
pin1.i + pin2.i = 0;
```

## Connection Equations

```
Pin pin1, pin2;  
//A connect equation  
//in Modelica  
connect(pin1, pin2);
```

Corresponds to

```
pin1.v = pin2.v;  
pin1.i + pin2.i = 0;
```

Multiple connections are possible:

```
connect(pin1, pin2); connect(pin1, pin3); ... connect(pin1, pinN);
```

Each primitive connection set of **nonflow** variables is used to generate equations of the form:

$$V_1 = V_2 = V_3 = \dots V_n$$

Each primitive connection set of **flow** variables is used to generate *sum-to-zero* equations of the form:

$$i_1 + i_2 + \dots (-i_k) + \dots i_n = 0$$

## Acausal, Causal, and Composite Connections

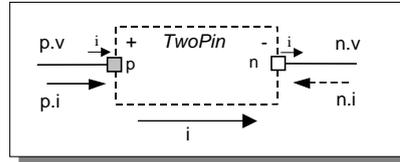
Two *basic* and one *composite* kind of connection in Modelica

- *Acausal connections*
- *Causal connections*, also called *signal connections*
- *Composite connections*, also called *structured connections*, composed of basic or composite connections

```
connector class ← connector OutPort  
fixed causality ← output Real signal;  
end OutPort
```

## Common Component Structure

The base class TwoPin has two connectors  $p$  and  $n$  for positive and negative pins respectively



partial class  
(cannot be  
instantiated)

positive pin

negative pin

```

partial model TwoPin
  Voltage v
  Current i
  Pin p;
  Pin n;
equation
  v = p.v - n.v;
  0 = p.i + n.i;
  i = p.i;
end TwoPin;
// TwoPin is same as OnePort in
// Modelica.Electrical.Analog.Interfaces
    
```

```

connector Pin
  Voltage v;
  flow Current i;
end Pin;
    
```

electrical connector class

## Electrical Components

```

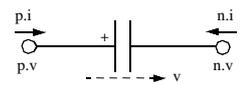
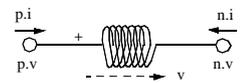
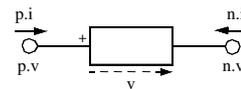
model Resistor "Ideal electrical resistor"
  extends TwoPin;
  parameter Real R;
equation
  R*i = v;
end Resistor;
    
```

```

model Inductor "Ideal electrical inductor"
  extends TwoPin;
  parameter Real L "Inductance";
equation
  L*der(i) = v;
end Inductor;
    
```

```

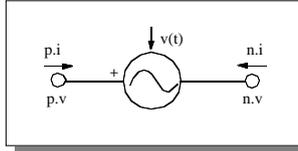
model Capacitor "Ideal electrical capacitor"
  extends TwoPin;
  parameter Real C ;
equation
  i=C*der(v) ;
end Inductor;
    
```



## Electrical Components cont'

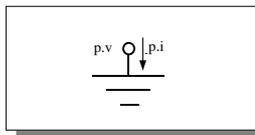
```

model Source
  extends TwoPin;
  parameter Real A,w;
  equation
    v = A*sin(w*time);
  end Resistor;
  
```

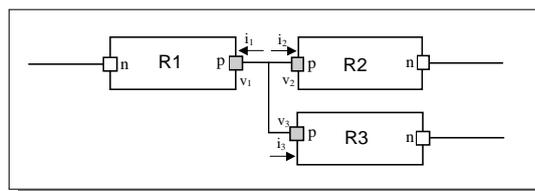


```

model Ground
  Pin p;
  equation
    p.v = 0;
  end Ground;
  
```



## Resistor Circuit



```

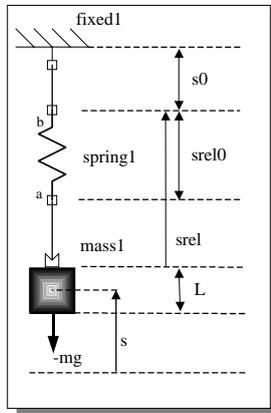
model ResistorCircuit
  Resistor R1(R=100);
  Resistor R2(R=200);
  Resistor R3(R=300);
  equation
    connect(R1.p, R2.p);
    connect(R1.n, R3.p);
  end ResistorCircuit;
  
```

Corresponds to

```

R1.p.v = R2.p.v;
R1.p.v = R3.p.v;
R1.p.i + R2.p.i + R3.p.i = 0;
  
```

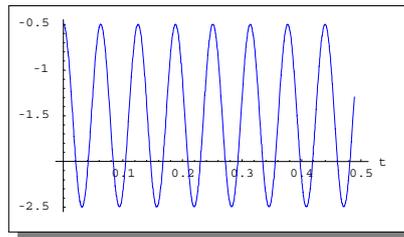
## An Oscillating Mass Connected to a Spring



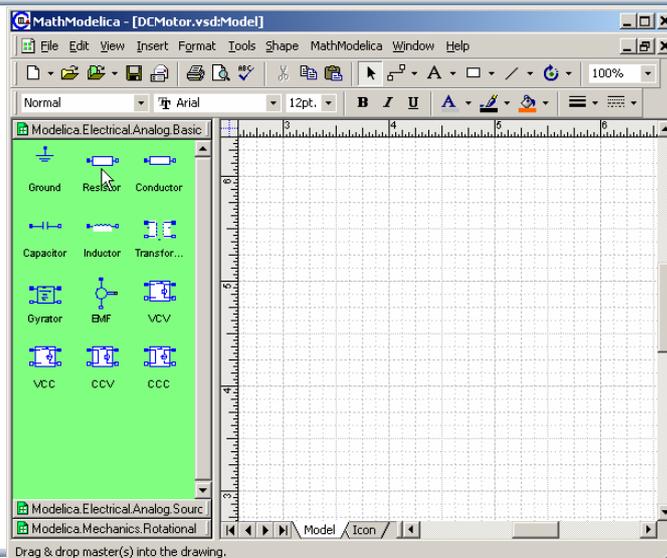
```

model Oscillator
  Mass   mass1(L=1, s(start=-0.5));
  Spring spring1(srel0=2, c=10000);
  Fixed  fixed1(s0=1.0);
equation
  connect(spring1.flange_b, fixed1.flange_b);
  connect(mass1.flange_b, spring1.flange_a);
end Oscillator;

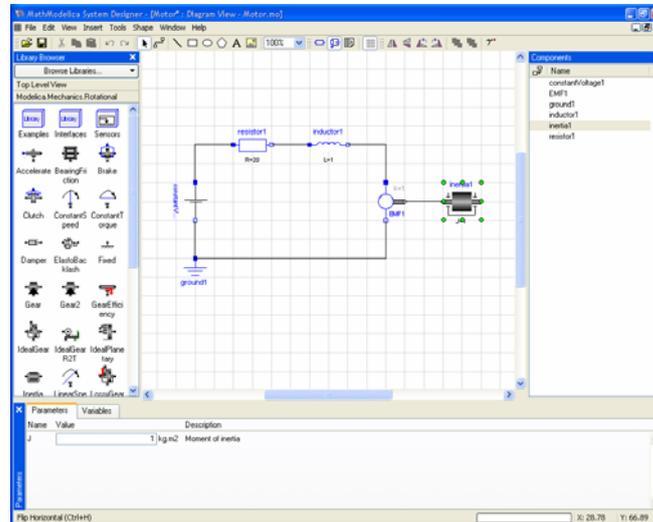
```



## Graphical Modeling Using Drag and Drop Composition



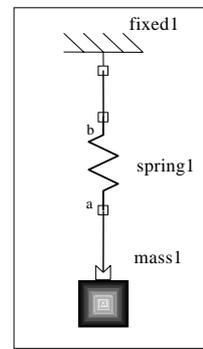
## Completed DCMotor using Graphical Composition



Courtesy MathCore Engineering AB

## Exercise

- Locate the Oscillator model in DrModelica using OMNotebook!
- Simulate and plot the example. Do a slight change in the model e.g. different elasticity  $c$ , re-simulate and re-plot.
- Draw the Oscillator model using the graphic connection editor e.g. using the library Modelica.Mechanics.Translational
- Including components SlidingMass, Force, Blocks.Sources.Constant
- Simulate and plot!



## Signal Based Connector Classes

```

connector InPort "Connector with input signals of type Real"
  parameter Integer n=1 "Dimension of signal vector";
  input Real signal[n] "Real input signals";
end InPort;

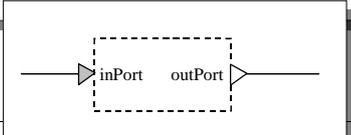
connector OutPort "Connector with output signals of type Real"
  parameter Integer n=1 "Dimension of signal vector";
  output Real signal[n] "Real output signals";
end OutPort;

partial block MISO
  "Multiple Input Single Output continuous control block"
  parameter Integer nin=1 "Number of inputs";
  InPort inPort(n=nin) "Connector of Real input signals";
  OutPort outPort(n=1) "Connector of Real output signal";
protected
  Real u[:] = inPort.signal "Input signals";
  Real y = outPort.signal[1] "Output signal";
end MISO; // From Modelica.Blocks.Interfaces
  
```

fixed causality ←

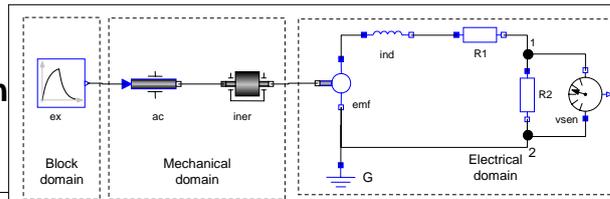
fixed causality ←

multiple input single output block



## Connecting Components from Multiple Domains

- Block domain
- Mechanical domain
- Electrical domain



### model Generator

```

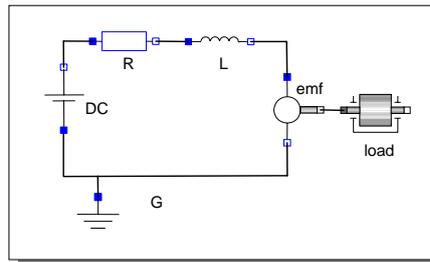
Modelica.Mechanics.Rotational.Accelerate ac;
Modelica.Mechanics.Rotational.Inertia iner;
Modelica.Electrical.Analog.Basic.EMF emf(k=-1);
Modelica.Electrical.Analog.Basic.Inductor ind(L=0.1);
Modelica.Electrical.Analog.Basic.Resistor R1,R2;
Modelica.Electrical.Analog.Basic.Ground G;
Modelica.Electrical.Analog.Sensors.VoltageSensor vsens;
Modelica.Blocks.Sources.Exponentials ex(riseTime={2},riseTimeConst={1});
equation
  connect(ac.flange_b, iner.flange_a); connect(iner.flange_b, emf.flange_b);
  connect(emf.p, ind.p); connect(ind.n, R1.p); connect(emf.n, G.p);
  connect(emf.n, R2.n); connect(R1.n, R2.p); connect(R2.p, vsens.n);
  connect(R2.n, vsens.p); connect(ex.outPort, ac.inPort);
end Generator;
  
```

## Simple Modelica DCMotor Model Multi-Domain (Electro-Mechanical)

A DC motor can be thought of as an electrical circuit which also contains an electromechanical component.

```

model DCMotor
  Resistor R(R=100);
  Inductor L(L=100);
  VsourceDC DC(f=10);
  Ground G;
  EMF emf(k=10,J=10, b=2);
  Inertia load;
equation
  connect(DC.p,R.n);
  connect(R.p,L.n);
  connect(L.p, emf.n);
  connect(emf.p, DC.n);
  connect(DC.n,G.p);
  connect(emf.flange,load.flange);
end DCMotor;
  
```

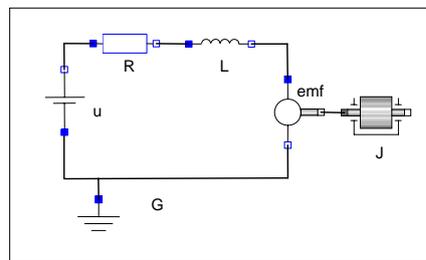


## Exercise

- Draw the `DCMotor` model using the graphic connection editor using models from the following Modelica libraries:

`Mechanics.Rotational`,  
`Electrical.Analog.Basic`,  
`Electrical.Analog.Sources`

- Simulate it for 15s and plot the variables for the outgoing rotational speed on the inertia axis and the voltage on the voltage source (denoted `u` in the figure) in the same plot.



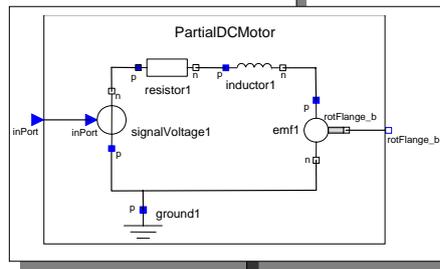
## Hierarchically Structured Components

An *inside connector* is a connector belonging to an *internal component* of a structured component class.

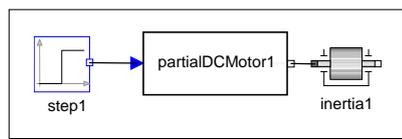
An *outside connector* is a connector that is part of the *external interface* of a structured component class, is declared directly within that class

```

partial model PartialDCMotor
  InPort      inPort; // Outside signal connector
  RotFlange_b rotFlange_b; // Outside rotational flange connector
  Inductor    inductor1;
  Resistor    resistor1;
  Ground      ground1;
  EMF         emf1;
  SignalVoltage signalVoltage1;
equation
  connect(inPort, signalVoltage1.inPort);
  connect(signalVoltage1.n, resistor1.p);
  connect(resistor1.n, inductor1.p);
  connect(signalVoltage1.p, ground1.p);
  connect(ground1.p, emf1.n);
  connect(inductor1.n, emf1.p);
  connect(emf1.rotFlange_b, rotFlange_b);
end PartialDCMotor;
  
```



## Hierarchically Structured Components cont'

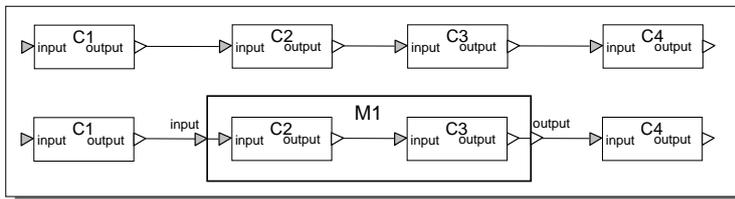


```

model DCMotorCircuit2
  Step      step1;
  PartialDCMotor partialDCMotor1;
  Inertia   inertia1;
equation
  connect(step1.outPort, partialDCMotor1.inPort);
  connect(partialDCMotor1.rotFlange_b, inertia1.rotFlange_a);
end DCMotorCircuit2;
  
```

## Connection Restrictions

- Two *acausal* connectors can be connected to each other
- An input connector can be connected to an output connector or vice versa
- An input or output connector can be connected to an *acausal* connector, i.e. a connector without input/output prefixes
- An *outside* input connector behaves approximately like an output connector internally
- An *outside* output connector behaves approximately like an input connector internally



## Connector Restrictions cont'

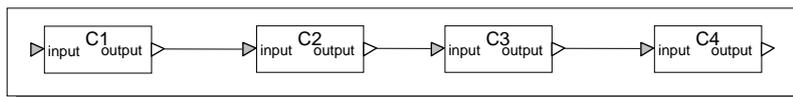
```
connector RealInput
  input Real signal;
end RealInput;
```

```
connector RealOutput
  output Real signal;
end RealOutput;
```

```
class C
  >RealInput u; // input connector
  >RealOutput y; // output connector
end C;
```

```
class CInst
  C C1, C2, C3, C4; // Instances of C
equation
  connect(C1.outPort, C2.inPort);
  connect(C2.outPort, C3.inPort);
  connect(C3.outPort, C4.inPort);
end CInst;
```

A circuit consisting of four connected components C1, C2, C3, and C4 which are instances of the class C

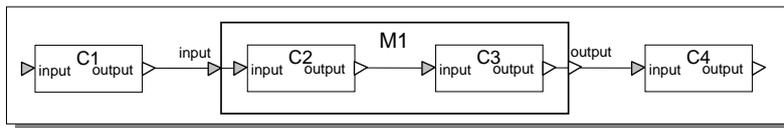


## Connector Restrictions cont'

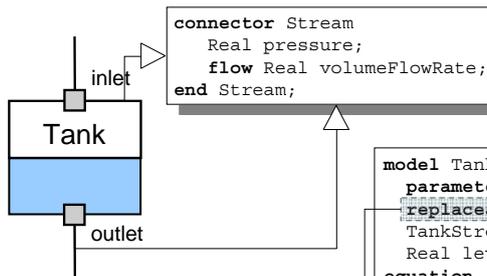
```
class M "Structured class M"
  RealInput u; // Outside input connector
  RealOutput y; // Outside output connector
  C C2;
  C C3;
end M;
```

A circuit in which the middle components C2 and C3 are placed inside a structured component M1 to which two outside connectors M1.u and M1.y have been attached.

```
class MInst
  M M1; // Instance of M
equation
  connect(C1.y, M1.u); // Normal connection of outPort to inPort
  connect(M1.u, C2.u); // Outside inPort connected to inside inPort
  connect(C2.y, C3.u); // Inside outPort connected to inside inPort
  connect(C3.y, M1.y); // Inside outPort connected to outside outPort
  connect(M1.y, C4.u); // Normal connection of outPort to inPort
end MInst;
```



## Parameterization and Extension of Interfaces

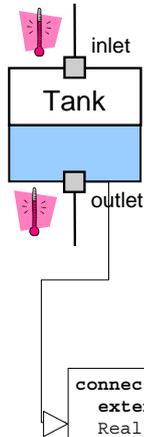


External interfaces to component classes are defined primarily through the use of connectors.

The Tank model has an external interface in terms of the connectors inlet and outlet

```
model Tank
  parameter Real Area=1;
  replaceable connector TankStream = Stream;
  TankStream inlet, outlet; // The connectors
  Real level;
equation
  // Mass balance
  Area*der(level) = inlet.volumeFlowRate +
    outlet.volumeFlowRate;
  outlet.pressure = inlet.pressure;
end Tank;
connector Stream // Connector class
  Real pressure;
  flow Real volumeFlowRate;
end Stream
```

## Parameterization and Extension of Interfaces – cont'



We would like to extend the Tank model to include temperature-dependent effects, analogous to how we extended a resistor to a temperature-dependent resistor

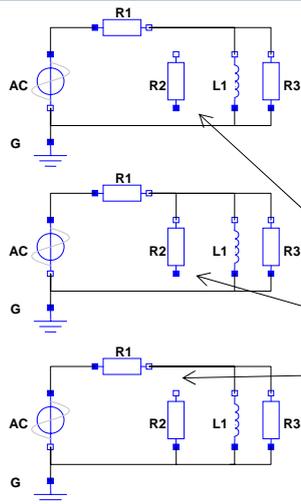
```

model HeatTank
  extends Tank(redeclare connector TankStream = HeatStream);
  Real temp;
  equation
    // Energy balance for temperature effects
    Area*level*der(temp) = inlet.volumeFlowRate*inlet.temp +
                          outlet.volumeFlowRate*outlet.temp;
    outlet.temp = temp; // Perfect mixing assumed.
  end HeatTank;
  
```

```

connector HeatStream
  extends Stream;
  Real temp;
  end HeatStream;
  
```

## Cardinality-dependent Connection Equations



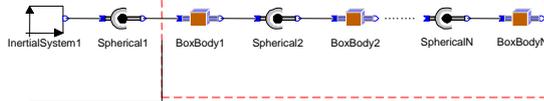
In certain cases there is a need to let the behavior of a model be dependent on the number of connections to certain connectors of the model. This can be achieved by using a built-in function `cardinality()` that returns the number of connections that have been made to a connector. (if-equations, see Lecture 4)

```

model CardinalityResistor
  extends TwoPin;
  parameter Real R(unit="Ohm") "Resistance";
  equation
    // Handle cases if pins are not connected
    if cardinality(p) == 0 and cardinality(n) == 0
      then p.v = 0; n.v = 0;
    elseif cardinality(p) == 0 then
      p.i = 0;
    elseif cardinality(n) == 0 then
      n.i = 0;
    end if
    // Resistor equation
    v = R*i;
  end CardinalityResistor;
  
```

## Arrays of Connectors

Part built up with a for-equation (see Lecture 4)



The model uses a for-equation to connect the different segments of the links

```
model ArrayOfLinks
  constant Integer n=10 "Number of segments (>0)";
  parameter Real[3,n] r={fill(1,n),zeros(n),zeros(n)};
  ModelicaAdditions.MultiBody.Parts.InertialSystem InertialSystem1;
  ModelicaAdditions.MultiBody.Parts.BoxBody[n]
    boxBody(r = r, Width=fill(0.4,n));
  ModelicaAdditions.MultiBody.Joints.Spherical spherical[n];
equation
  connect(InertialSystem1.frame_b, spherical[1].frame_a);
  connect(spherical[1].frame_b, boxBody[1].frame_a);
  for i in 1:n-1 loop
    connect(boxBody[i].frame_b, spherical[i+1].frame_a);
    connect(spherical[i+1].frame_b, boxBody[i+1].frame_a);
  end for;
end ArrayOfLinks;
```

# Equations, Algorithms, and Functions

## Equations

## Usage of Equations

In Modelica equations are used for many tasks

- The main usage of equations is to represent relations in mathematical models.
- *Assignment statements* in conventional languages are usually represented as equations in Modelica
- *Attribute assignments* are represented as equations
- Connections between objects generate equations

## Equation Categories

Equations in Modelica can informally be classified into three different categories

- *Normal equations* (e.g.,  $expr1 = expr2$ ) occurring in equation sections, including `connect` equations and other equation types of special syntactic form
- *Declaration equations*, (e.g., `Real x = 2.0`) which are part of variable, parameter, or constant declarations
- *Modifier equations*, (e.g. `x(unit="V")`) which are commonly used to modify attributes of classes.

## Constraining Rules for Equations

### Single Assignment Rule

The total number of “equations” is identical to the total number of “unknown” variables to be solved for

### Synchronous Data Flow Principle

- All variables keep their actual values until these values are explicitly changed
- At every point in time, during “continuous integration” and at event instants, the *active* equations express relations between variables which have to be fulfilled *concurrently*  
Equations are not active if the corresponding `if`-branch or `when`-equation in which the equation is present is not active because the corresponding branch condition currently evaluates to `false`
- Computation and communication at an event instant does not take time

## Declaration Equations

Declaration equations:

```
constant Integer one = 1;  
parameter Real mass = 22.5;
```

It is also possible to specify a declaration equation for a normal non-constant variable:

```
Real speed = 72.4;
```

declaration equations

```
model MoonLanding  
  parameter Real force1 = 36350;  
  parameter Real force2 = 1308;  
  parameter Real thrustEndTime = 210;  
  parameter Real thrustDecreaseTime = 43.2;  
  Rocket      apollo(name="apollo13", mass(start=1038.358) );  
  CelestialBody moon(mass=7.382e22, radius=1.738e6, name="moon");  
equation  
  apollo.thrust = if (time<thrustDecreaseTime) then force1  
                  else if (time<thrustEndTime) then force2  
                  else 0;  
  apollo.gravity=moon.g*moon.mass/(apollo.altitude+moon.radius)^2;  
end Landing;
```

## Modifier Equations

Modifier equations occur for example in a variable declaration when there is a need to modify the default value of an attribute of the variable  
A common usage is modifier equations for the start attribute of variables

```
Real speed(start=72.4);
```

Modifier equations also occur in type definitions:

```
type Voltage = Real(unit="V", min=-220.0, max=220.0);
```

modifier equations

```
model MoonLanding  
  parameter Real force1 = 36350;  
  parameter Real force2 = 1308;  
  parameter Real thrustEndTime = 210;  
  parameter Real thrustDecreaseTime = 43.2;  
  Rocket      apollo(name="apollo13", mass(start=1038.358) );  
  CelestialBody moon(mass=7.382e22, radius=1.738e6, name="moon");  
equation  
  apollo.thrust = if (time<thrustDecreaseTime) then force1  
                  else if (time<thrustEndTime) then force2  
                  else 0;  
  apollo.gravity=moon.g*moon.mass/(apollo.altitude+moon.radius)^2;  
end Landing;
```

## Kinds of Normal Equations in Equation Sections

Kinds of equations that can be present in equation sections:

- equality equations
- connect equations
- assert and terminate
- reinit
- repetitive equation structures with for-equations
- conditional equations with if-equations
- conditional equations with when-equations

```
model MoonLanding
  parameter Real force1 = 36350;
  parameter Real force2 = 1308;
  parameter Real thrustEndTime = 210;
  parameter Real thrustDecreaseTime = 43.2;
  Rocket      apollo(name="apollo13", mass(start=1038.358) );
  CelestialBody moon(mass=7.382e22, radius=1.738e6, name="moon");

  equation
    if (time<thrustDecreaseTime) then
      apollo.thrust = force1;
    elseif (time<thrustEndTime) then
      apollo.thrust = force2;
    else
      apollo.thrust = 0;
    end if;
    apollo.gravity=moon.g*moon.mass/(apollo.altitude+moon.radius)^2;
end Landing;
```

conditional if-equation ←

equality equation ←

## Equality Equations

```
expr1 = expr2:
(out1, out2, out3, ...) = function_name(in_expr1, in_expr2, ...);
```

```
class EqualityEquations
  Real x,y,z;
  equation
    (x, y, z) = f(1.0, 2.0); // Correct!
    (x+1, 3.0, z/y) = f(1.0, 2.0); // Illegal!
    // Not a list of variables
    // on the left-hand side
end EqualityEquations;
```

simple equality equation ←

## Repetitive Equations

The syntactic form of a `for`-equation is as follows:

```
for <iteration-variable> in <iteration-set-expression> loop
  <equation1>
  <equation2>
  ...
end for;
```

Consider the following simple example with a `for`-equation:

```
class FiveEquations
  Real[5] x;
  equation
    for i in 1:5 loop
      x[i] = i+1;
    end for;
end FiveEquations;
```

**Both classes have  
equivalent behavior!**

```
class FiveEquationsUnrolled
  Real[5] x;
  equation
    x[1] = 2;
    x[2] = 3;
    x[3] = 4;
    x[4] = 5;
    x[5] = 6;
end FiveEquationsUnrolled;
```

In the class on the right the `for`-equation has been unrolled into five simple equations

## connect-equations

In Modelica `connect`-equations are used to establish connections between components via connectors

```
connect(connector1, connector2)
```

Repetitive `connect`-equations

```
class RegComponent
  Component components[n];
  equation
    for i in 1:n-1 loop
      connect(components[i].outlet, components[i+1].inlet);
    end for;
end RegComponent;
```

## Conditional Equations: if-equations

```
if <condition> then
  <equations>
elseif <condition> then
  <equations>
else
  <equations>
end if;
```

*if-equations* for which the conditions have higher variability than constant or parameter must include an *else-part*

Each then-, elseif-, and else-branch must have the *same number of equations*

```
model MoonLanding
  parameter Real force1 = 36350;
  ...
  Rocket      apollo(name="apollo13", mass(start=1038.358) );
  CelestialBody moon(mass=7.382e22, radius=1.738e6, name="moon");
equation
  if (time<thrustDecreaseTime) then
    apollo.thrust = force1;
  elseif (time<thrustEndTime) then
    apollo.thrust = force2;
  else
    apollo.thrust = 0;
  end if;
  apollo.gravity=moon.g*moon.mass/(apollo.altitude+moon.radius)^2;
end Landing;
```

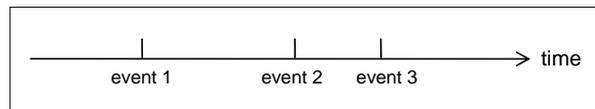
## Conditional Equations: when-equations

```
when <conditions> then
  <equations>
end when;
```

```
when x > 2 then
  y1 = sin(x);
  y3 = 2*x + y1+y2;
end when;
```

<equations> in when-equations are instantaneous equations that are active at events when <conditions> become true

Events are ordered in time and form an event history:



- An event is a *point* in time that is instantaneous, i.e., has zero duration
- An *event condition* switches from false to true in order for the event to take place

## Conditional Equations: when-equations cont'

```
when <conditions> then
  <equations>
end when;
```

*when-equations* are used to express instantaneous equations that are only valid (become active) *at events*, e.g. at discontinuities or when certain conditions become true

```
when x > 2 then
  y1 = sin(x);
  y3 = 2*x + y1+y2;
end when;
```

```
when {x > 2, sample(0,2), x < 5} then
  y1 = sin(x);
  y3 = 2*x + y1+y2;
end when;
```

```
when initial() then
  ... // Equations to be activated at the beginning of a simulation
end when;
...
when terminal() then
  ... // Equations to be activated at the end of a simulation
end when;
```

## Restrictions on when-equations

### Form restriction

```
model WhenNotValid
  Real x, y;
  equation
    x + y = 5;
  when sample(0,2) then
    2*x + y = 7;
  // Error: not valid Modelica
  end when;
end WhenNotValid;
```

Modelica restricts the allowed equations within a when-equation to: **variable = expression**, if-equations, for-equations,...

In the WhenNotValid model when the equations within the when-equation are not active it is not clear which variable, either x or y, that is a “result” from the when-equation to keep constant outside the when-equation.

A corrected version appears in the class WhenValidResult below

```
model WhenValidResult
  Real x,y;
  equation
    x + y = 5; // Equation to be used to compute x.
  when sample(0,2) then
    y = 7 - 2*x; // Correct, y is a result variable from the when;
  end when;
end WhenValidResult;
```

## Restrictions on when-equations cont'

### Restriction on nested when-equations

```
model ErrorNestedWhen
  Real x,y1,y2;
  equation
    when x > 2 then
      when y1 > 3 then // Error!
        y2 = sin(x); // when-equations
      end when; // should not be nested
    end when;
end ErrorNestedWhen;
```

when-equations cannot be nested!

## Restrictions on when-equations cont'

Single assignment rule: same variable may not be defined in several when-equations.

A conflict between the equations will occur if both conditions would become true at the same time instant

```
model DoubleWhenConflict
  Boolean close; // Error: close defined by two equations!
  equation
    ...
    when condition1 then
      close = true; // First equation
    end when;
    ...
    when condition2 then
      close = false; // Second equation
    end when;
  end DoubleWhenConflict
```

## Restrictions on when-equations cont'

Solution to assignment conflict between equations in independent when-equations:

- Use `elsewhen` to give higher priority to the first when-equation

```
model DoubleWhenConflictResolved
  Boolean close;
equation
  ...
  when condition1 then
    close = true; // First equation has higher priority!
  elsewhen condition2 then
    close = false; // Second equation
  end when;
end DoubleWhenConflictResolved
```

## Restrictions on when-equations cont'

Vector expressions

The equations within a when-equation are activated when any of the elements of the vector expression becomes true

```
model VectorWhen
  Boolean close;
equation
  ...
  when {condition1, condition2} then
    close = true;
  end when;
end DoubleWhenConflict
```

## assert-equations

```
assert(assert-expression, message-string)
```

`assert` is a predefined function for giving error messages taking a Boolean condition and a string as an argument

The intention behind `assert` is to provide a convenient means for specifying checks on model validity within a model

```
class AssertTest
  parameter Real lowlimit = -5;
  parameter Real highlimit = 5;
  Real x;
equation
  assert(x >= lowlimit and x <= highlimit,
         "Variable x out of limit");
end AssertTest;
```

## terminate-equations

The `terminate-equation` successfully terminates the current simulation, i.e. no error condition is indicated

```
model MoonLanding
  parameter Real force1 = 36350;
  parameter Real force2 = 1308;
  parameter Real thrustEndTime = 210;
  parameter Real thrustDecreaseTime = 43.2;
  Rocket apollo(name="apollo13", mass(start=1038.358) );
  CelestialBody moon(mass=7.382e22, radius=1.738e6, name="moon");
equation
  apollo.thrust = if (time<thrustDecreaseTime) then force1
                  else if (time<thrustEndTime) then force2
                  else 0;
  apollo.gravity = moon.g * moon.mass / (apollo.height + moon.radius)^2;
  when apollo.height < 0 then // termination condition
    terminate("The moon lander touches the ground of the moon");
  end when;
end MoonLanding;
```

# Algorithms and Functions

## Algorithm Sections

Whereas equations are very well suited for physical modeling, there are situations where computations are more conveniently expressed as algorithms, i.e., sequences of instructions, also called statements

```
algorithm
...
<statements>
...
<some keyword>
```

Algorithm sections can be embedded among equation sections

```
equation
  x = y*2;
  z = w;
algorithm
  x1 := z+x;
  x2 := y-5;
  x1 := x2+y;
equation
  u = x1+x2;
  ...
```

## Iteration Using for-statements in Algorithm Sections

```
for <iteration-variable> in <iteration-set-expression> loop
  <statement1>
  <statement2>
  ...
end for
```

The general structure of a for-statement with a single iterator

```
class SumZ
  parameter Integer n = 5;
  Real[n] z(start = {10,20,30,40,50});
  Real sum;
  algorithm
    sum := 0;
    for i in 1:n loop // 1:5 is {1,2,3,4,5}
      sum := sum + z[i];
    end for;
  end SumZ;
```

A simple for-loop summing the five elements of the vector z, within the class SumZ

Examples of for-loop headers with different range expressions

```
for k in 1:10+2 loop // k takes the values 1,2,3,...,12
for i in {1,3,6,7} loop // i takes the values 1, 3, 6, 7
for r in 1.0 : 1.5 : 5.5 loop // r takes the values 1.0, 2.5, 4.0, 5.5
```

## Iterations Using while-statements in Algorithm Sections

```
while <conditions> loop
  <statements>
end while;
```

The general structure of a while-loop with a single iterator.

```
class SumSeries
  parameter Real eps = 1.E-6;
  Integer i;
  Real sum;
  Real delta;
  algorithm
    i := 1;
    delta := exp(-0.01*i);
    while delta>=eps loop
      sum := sum + delta;
      i := i+1;
      delta := exp(-0.01*i);
    end while;
  end SumSeries;
```

The example class SumSeries shows the while-loop construct used for summing a series of exponential terms until the loop condition is violated, i.e., the terms become smaller than eps.

## if-statements

```
if <condition> then
  <statements>
elseif <condition> then
  <statements>
else
  <statements>
end if
```

The if-statements used in the class SumVector perform a combined summation and computation on a vector  $v$ .

The general structure of if-statements. The elseif-part is optional and can occur zero or more times whereas the optional else-part can occur at most once

```
class SumVector
  Real sum;
  parameter Real v[5] = {100,200,-300,400,500};
  parameter Integer n = size(v,1);
algorithm
  sum := 0;
  for i in 1:n loop
    if v[i]>0 then
      sum := sum + v[i];
    elseif v[i] > -1 then
      sum := sum + v[i] -1;
    else
      sum := sum - v[i];
    end if;
  end for;
end SumVector;
```

## when-statements

```
when <conditions> then
  <statements>
elsewhen <conditions> then
  <statements>
end when;
```

when-statements are used to express actions (statements) that are only executed at events, e.g. at discontinuities or when certain conditions become true

There are situations where several assignment statements within the same when-statement is convenient

```
when x > 2 then
  y1 := sin(x);
  y3 := 2*x + y1 + y2;
end when;
```

```
algorithm
  when x > 2 then
    y1 := sin(x);
  end when;
equation
  y2 = sin(y1);
algorithm
  when x > 2 then
    y3 := 2*x + y1 + y2;
  end when;
```

```
when {x > 2, sample(0,2), x < 5} then
  y1 := sin(x);
  y3 := 2*x + y1 + y2;
end when;
```

Algorithm and equation sections can be interleaved.

## Function Declaration

The structure of a typical function declaration is as follows:

```
function <functionname>
input Type1 in1;
input Type2 in2;
input Type3 in3;
...
output TypeO1 out1;
output TypeO2 out2;
...
protected
<local variables>
...
algorithm
<statements>
...
end <functionname>;
```

All internal parts of a function are optional, the following is also a legal function:

```
function <functionname>
end <functionname>;
```

Modelica functions are *declarative mathematical functions*:

- Always return the same result(s) given the same input argument values

## Function Call

Two basic forms of arguments in Modelica function calls:

- *Positional* association of actual arguments to formal parameters
- *Named* association of actual arguments to formal parameters

Example function called on next page:

```
function PolynomialEvaluator
input Real A[1:n] // array, size defined
// at function call time
input Real x := 1.0 // default value 1.0 for x
output Real sum;
protected
Real xpower; // local variable xpower
algorithm
sum := 0;
xpower := 1;
for i in 1:size(A,1) loop
sum := sum + A[i]*xpower;
xpower := xpower*x;
end for;
end PolynomialEvaluator;
```

The function PolynomialEvaluator computes the value of a polynomial given two arguments: a coefficient vector A and a value of x.

## Positional and Named Argument Association

Using *positional* association, in the call below the actual argument  $\{1, 2, 3, 4\}$  becomes the value of the coefficient vector  $A$ , and 21 becomes the value of the formal parameter  $x$ .

```
...
algorithm
...
p:= polynomialEvaluator({1,2,3,4},21)
```

The same call to the function `polynomialEvaluator` can instead be made using *named* association of actual parameters to formal parameters.

```
...
algorithm
...
p:= polynomialEvaluator(A={1,2,3,4},x=21)
```

## Functions with Multiple Results

```
function PointOnCircle"Computes cartesian coordinates of point"
input Real angle "angle in radians";
input Real radius;
output Real x; // 1:st result formal parameter
output Real y; // 2:nd result formal parameter
algorithm
x := radius * cos(phi);
y := radius * sin(phi);
end PointOnCircle;
```

Example calls:

```
(out1,out2,out3,...) = function_name(in1, in2, in3, in4, ...); // Equation
(out1,out2,out3,...) := function_name(in1, in2, in3, in4, ...); // Statement

(px,py) = PointOnCircle(1.2, 2); // Equation form
(px,py) := PointOnCircle(1.2, 2); // Statement form
```

Any kind of variable of compatible type is allowed in the parenthesized list on the left hand side, e.g. even array elements:

```
(arr[1],arr[2]) := PointOnCircle(1.2, 2);
```

## External Functions

It is possible to call functions defined outside the Modelica language, implemented in C or FORTRAN 77

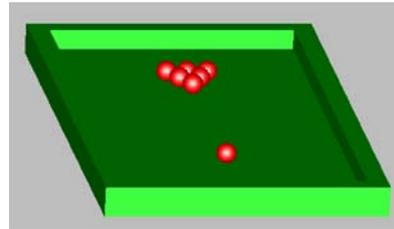
```
function polynomialMultiply
  input Real a[:], b[:];
  output Real c[:] := zeros(size(a,1)+size(b, 1) - 1);
  external
end polynomialMultiply;
```

The body of an external function is marked with the keyword **external**

If no language is specified, the implementation language for the external function is assumed to be C. The external function `polynomialMultiply` can also be specified, e.g. via a mapping to a FORTRAN 77 function:

```
function polynomialMultiply
  input Real a[:], b[:];
  output Real c[:] := zeros(size(a,1)+size(b, 1) - 1);
  external "FORTRAN 77"
end polynomialMultiply;
```

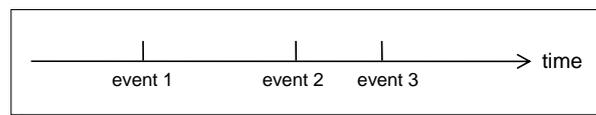
# Discrete Events and Hybrid Systems



Picture: Courtesy Håkan Elmquist

## Events

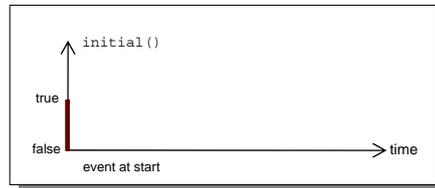
Events are ordered in time and form an event history



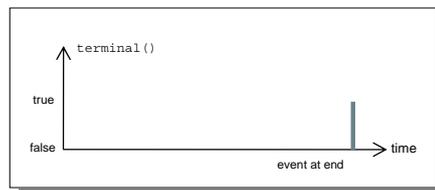
- A *point* in time that is instantaneous, i.e., has zero duration
- An *event condition* that switches from false to true in order for the event to take place
- A set of *variables* that are associated with the event, i.e. are referenced or explicitly changed by equations associated with the event
- Some *behavior* associated with the event, expressed as *conditional equations* that become active or are deactivated at the event.  
*Instantaneous equations* is a special case of conditional equations that are only active *at* events.

## initial and terminal events

Initialization actions are triggered by `initial()`

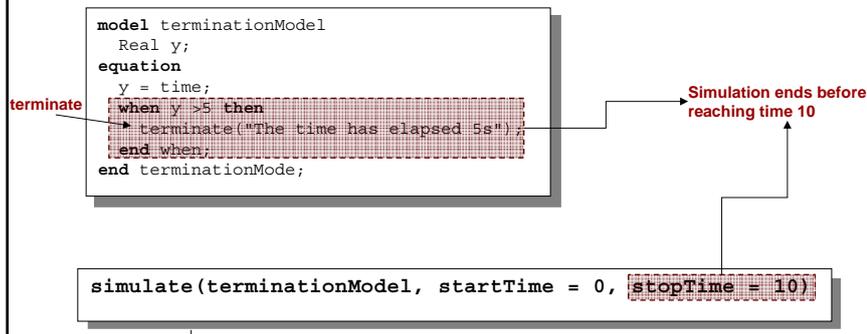


Actions at the end of a simulation are triggered by `terminal()`



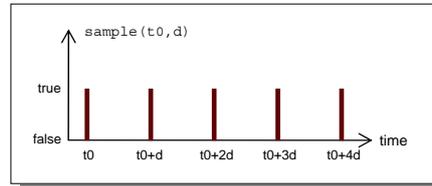
## Terminating a Simulation

The `terminate()` function is useful when a wanted result is achieved and it is no longer useful to continue the simulation. The example below illustrates the use:



## Generating Repeated Events

The call `sample(t0,d)` returns true and triggers events at times  $t_0+i*d$ , where  $i=0,1,\dots$



```
class SamplingClock
  parameter Modelica.SIunits.Time first, interval;
  Boolean clock;
  equation
    clock = sample(first, interval);
  when clock then
    ...
  end when;
end SamplingClock;
```

## Expressing Event Behavior in Modelica

*if-equations, if-statements, and if-expressions* express different behavior in different operating regions

```
if <condition> then
  <equations>
elseif <condition> then
  <equations>
else
  <equations>
end if;
```

```
model Diode "Ideal diode"
  extends TwoPin;
  Real s;
  Boolean off;
  equation
    off = s < 0;
    if off then
      v=s
    else
      v=0;
    end if;
    i = if off then 0 else s;
  end Diode;
```

*when-equations* become active at events

```
when <conditions> then
  <equations>
end when;
```

```
equation
  when x > y.start then
    ...
```

## Event Priority

Erroneous multiple definitions, single assignment rule violated

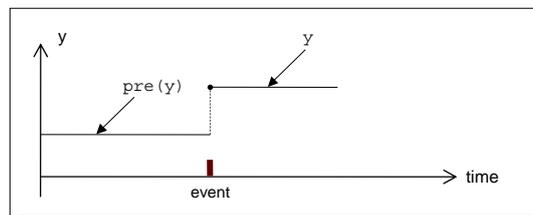
```
model WhenConflictX // Erroneous model: two equations define x
  discrete Real x;
  equation
  when time==2 then // When A: Increase x by 1.5 at time=2
    x = pre(x)+1.5;
  end when;
  when time==1 then // When B: Increase x by 1 at time=1
    x = pre(x)+1;
  end when;
end WhenConflictX;
```

Using event priority  
to avoid erroneous  
multiple definitions

```
model WhenPriorityX
  discrete Real x;
  equation
  when time==2 then // Higher priority
    x = pre(x)+1.5;
  elseif time==1 then // Lower priority
    x = pre(x)+1;
  end when;
end WhenPriorityX;
```

## Obtaining Predecessor Values of a Variable Using `pre()`

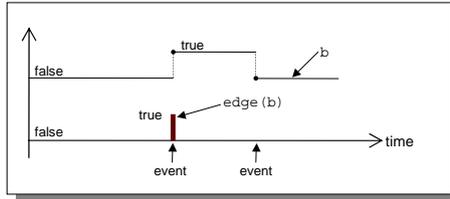
At an event, `pre(y)` gives the previous value of `y` immediately before the event, except for event iteration of multiple events at the same point in time when the value is from the previous iteration



- The variable `y` has one of the basic types Boolean, Integer, Real, String, or enumeration, a subtype of those, or an array type of one of those basic types or subtypes
- The variable `y` is a discrete-time variable
- The `pre` operator can *not* be used within a function

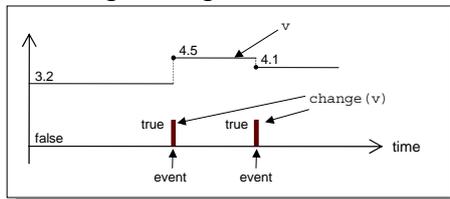
## Detecting Changes of Boolean Variables Using `edge()` and `change()`

Detecting changes of boolean variables using `edge()`



The expression `edge(b)` is true at events when `b` switches from false to true

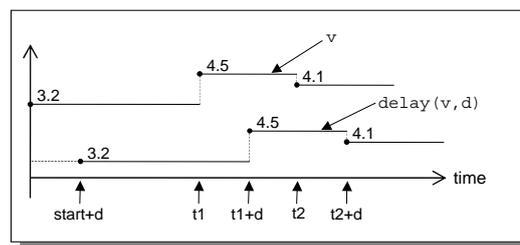
Detecting changes of discrete-time variables using `change()`



The expression `change(v)` is true at instants when `v` changes value

## Creating Time-Delayed Expressions

Creating time-delayed expressions using `delay()`



In the expression `delay(v, d)` `v` is delayed by a delay time `d`

## A Sampler Model

```

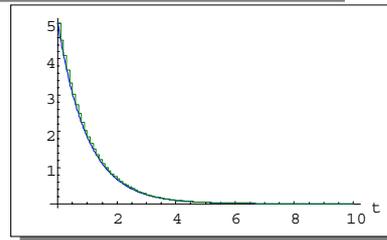
model Sampler
  parameter Real sample_interval = 0.1;
  Real x(start=5);
  Real y;
  equation
    der(x) = -x;
    when sample(0, sample_interval) then
      y = x;
    end when;
  end Sampler;

```

```

simulate(Sampler, startTime = 0, stopTime = 10)
plot({x,y})

```



## Discontinuous Changes to Variables at Events via When-Equations/Statements

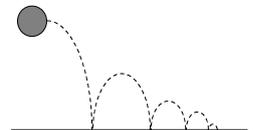
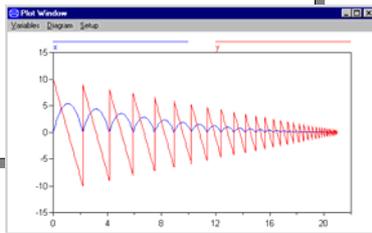
The value of a *discrete-time* variable can be changed by placing the variable on the left-hand side in an equation within a *when*-equation, or on the left-hand side of an assignment statement in a *when*-statement

The value of a *continuous-time* state variable can be instantaneously changed by a *reinit*-equation within a *when*-equation

```

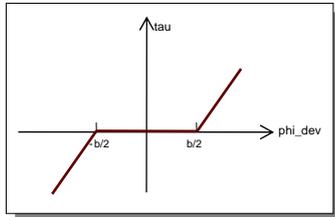
model BouncingBall "the bouncing ball model"
  parameter Real g=9.18; //gravitational acc.
  parameter Real c=0.90; //elasticity constant
  Real x(start=0), y(start=10);
  equation
    der(x) = y;
    der(y) = -g;
    when x<0 then
      reinit(y, -c*y);
    end when;
  end BouncingBall;

```

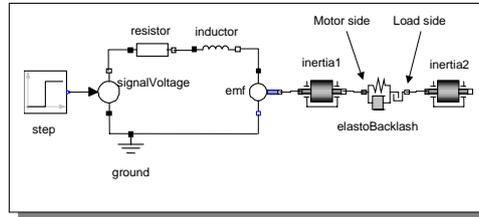


## A Mode Switching Model Example

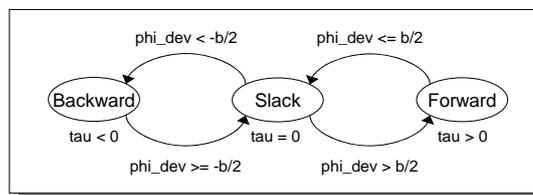
Elastic transmission with slack



DC motor transmission with elastic backlash



A finite state automaton  
SimpleElastoBacklash  
model



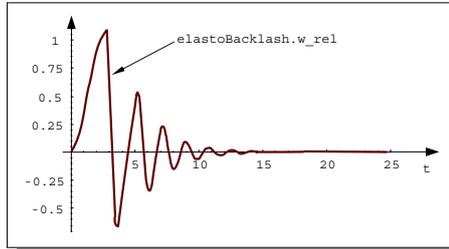
## A Mode Switching Model Example cont'

```

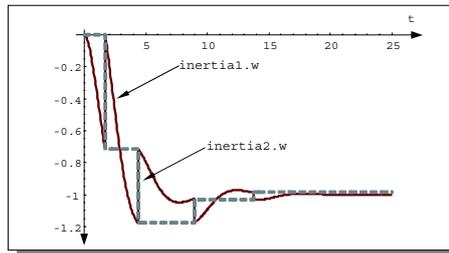
partial model SimpleElastoBacklash
  Boolean backward, slack, forward; // Mode variables
  parameter Real b "Size of backlash region";
  parameter Real c = 1.e5 "Spring constant (c>0), N.m/rad";
  Flange_a flange_a "(left) driving flange - connector";
  Flange_b flange_b "(right) driven flange - connector";
  parameter Real phi_rel0 = 0 "Angle when spring exerts no torque";
  Real phi_rel "Relative rotation angle betw. flanges";
  Real phi_dev "Angle deviation from zero-torque pos";
  Real tau "Torque between flanges";
equation
  phi_rel = flange_b.phi - flange_a.phi;
  phi_dev = phi_rel - phi_rel0;
  backward = phi_rel < -b/2; // Backward angle gives torque tau<0
  forward = phi_rel > b/2; // Forward angle gives torque tau>0
  slack = not (backward or forward); // Slack angle gives no torque
  tau = if forward then // Forward angle gives
    c*(phi_dev - b/2) // positive driving torque
  else if backward then // Backward angle gives
    c*(phi_dev + b/2) // negative braking torque
  else // Slack gives
    0; // zero torque
end SimpleElastoBacklash

```

## A Mode Switching Model Example cont'



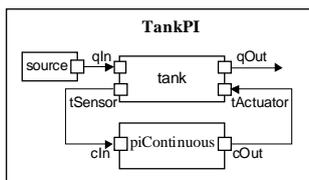
Relative rotational speed between the flanges of the Elastobacklash transmission



We define a model with less mass in inertia2 ( $J=1$ ), no damping  $d=0$ , and weaker spring constant  $c=1e-5$ , to show even more dramatic backlash phenomena

The figure depicts the rotational speeds for the two flanges of the transmission with elastic backlash

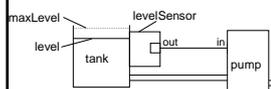
## Water Tank System with PI Controller



```

model TankPI
  LiquidSource      source(flowLevel=0.02);
  Tank              tank(area=1);
  PIcontinuousController piContinuous(ref=0.25);
equation
  connect(source.qOut, tank.qIn);
  connect(tank.tActuator, piContinuous.cOut);
  connect(tank.tSensor, piContinuous.cIn);
end TankPI;

```



```

model Tank
  ReadSignal tOut; // Connector, reading tank level
  ActSignal tInp; // Connector, actuator controlling input flow
  parameter Real flowVout = 0.01; // [m3/s]
  parameter Real area = 0.5; // [m2]
  parameter Real flowGain = 10; // [m2/s]
  Real h(start=0); // tank level [m]
  Real qIn; // flow through input valve[m3/s]
  Real qOut; // flow through output valve[m3/s]
equation
  der(h)=(qIn-qOut)/area; // mass balance equation
  qOut=if time>100 then flowVout else 0;
  qIn = flowGain*tInp.act;
  tOut.val = h;
end Tank;

```

## Water Tank System with PI Controller – cont'

```

partial model BaseController
  parameter Real Ts(unit = "s") = 0.1 "Time period between discrete samples";
  parameter Real K = 2 "Gain";
  parameter Real T(unit = "s") = 10 "Time constant";
  ReadSignal cIn "Input sensor level, connector";
  ActSignal cOut "Control to actuator, connector";
  parameter Real ref "Reference level";
  Real error "Deviation from reference level";
  Real outCtr "Output control signal";
equation
  error = ref - cIn.val;
  cOut.act = outCtr;
end BaseController;

```

```

model PIDContinuousController
  extends BaseController(K = 2, T = 10);
  Real x;
  Real y;
equation
  der(x) = error/T;
  y = T*der(error);
  outCtr = K*(error + x + y);
end PIDContinuousController;

```

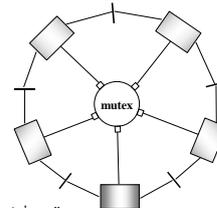
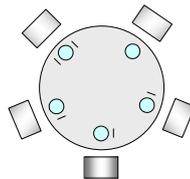
```

model PIdiscreteController
  extends BaseController(K = 2, T = 10);
  discrete Real x;
equation
  when sample(0, Ts) then
    x = pre(x) + error * Ts / T;
    outCtr = K * (x+error);
  end when;
end PIdiscreteController;

```

## Concurrency and Resource Sharing

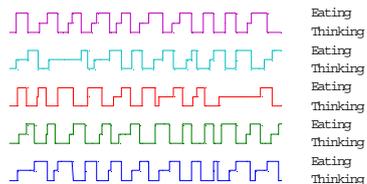
### Dining Philosophers Example



```

model DiningTable
  parameter Integer n = 5 "Number of philosophers and forks";
  parameter Real sigma = 5 "Standard deviation for the random function";
  // Give each philosopher a different random start seed
  // Comment out the initializer to make them all hungry simultaneously.
  Philosopher phil[n](startSeed=[1:n,1:n,1:n], sigma=fill(sigma,n));
  Mutex mutex(n=n);
  Fork fork[n];
equation
  for i in 1:n loop
    connect(phil[i].mutexPort, mutex.port[i]);
    connect(phil[i].right, fork[i].left);
    connect(fork[i].right, phil[mod(i, n) + 1].left);
  end for;
end DiningTable;

```



# Packages

## Packages for Avoiding Name Collisions

- Modelica provide a safe and systematic way of avoiding name collisions through the package concept
- A package is simply a container or name space for names of classes, functions, constants and other allowed definitions

## Packages as Abstract Data Type: Data and Operations in the Same Place

Keywords denoting a package

encapsulated package makes package dependencies (i.e., imports) explicit

Declarations of substract, divide, realPart, imaginaryPart, etc are not shown here

```

encapsulated package ComplexNumber

  record Complex
    Real re;
    Real im;
  end Complex;

  function add
    input Complex x,y;
    output Complex z;
  algorithm
    z.re := x.re + y.re;
    z.im := x.im + y.im;
  end add;

  function multiply
    input Complex x,y;
    output Complex z;
  algorithm
    z.re := x.re*y.re - x.im*y.im;
    z.im := x.re*y.im + x.im*y.re;
  end multiply;
end ComplexNumbers
    
```

Usage of the ComplexNumber package

```

class ComplexUser
  ComplexNumbers.Complex a(re=1.0, im=2.0);
  ComplexNumbers.Complex b(re=1.0, im=2.0);
  ComplexNumbers.Complex z,w;
equation
  z = ComplexNumbers.multiply(a,b);
  w = ComplexNumbers.add(a,b);
end ComplexUser
    
```

The type Complex and the operations multiply and add are referenced by prefixing them with the package name ComplexNumber

## Accessing Definitions in Packages

- Access reference by prefixing the package name to definition names

```

class ComplexUser
  ComplexNumbers.Complex a(re=1.0, im=2.0);
  ComplexNumbers.Complex b(re=1.0, im=2.0);
  ComplexNumbers.Complex z,w;
equation
  z = ComplexNumbers.multiply(a,b);
  w = ComplexNumbers.add(a,b);
end ComplexUser
    
```

- Shorter access names (e.g. Complex, multiply) can be used if definitions are first imported from a package (see next page).

## Importing Definitions from Packages

• Qualified import	←	<code>import &lt;packagename&gt;</code>
• Single definition import	←	<code>import &lt;packagename&gt; . &lt;definitionname&gt;</code>
• Unqualified import	←	<code>import &lt;packagename&gt; . *</code>
• Renaming import	←	<code>import &lt;shortpackagename&gt; = &lt;packagename&gt;</code>

The four forms of import are exemplified below assuming that we want to access the addition operation (add) of the package `Modelica.Math.ComplexNumbers`

```
import Modelica.Math.ComplexNumbers; //Access as ComplexNumbers.add
import Modelica.Math.ComplexNumbers.add; //Access as add
import Modelica.Math.ComplexNumbers.* //Access as add
import Co = Modelica.Math.ComplexNumbers //Access as Co.add
```

## Qualified Import

Qualified import ← `import <packagename>`

The *qualified import* statement `import <packagename>;` imports all definitions in a package, which subsequently can be referred to by (usually shorter) names *simplepackagename . definitionname*, where the simple package name is the *packagename* without its prefix.

```
encapsulated package ComplexUser1
import Modelica.Math.ComplexNumbers;
class User
  ComplexNumbers.Complex a(x=1.0, y=2.0);
  ComplexNumbers.Complex b(x=1.0, y=2.0);
  ComplexNumbers.Complex z, w;
equation
  z = ComplexNumbers.multiply(a, b);
  w = ComplexNumbers.add(a, b);
end User;
end ComplexUser1;
```

This is the most common form of import that eliminates the risk for name collisions when importing from several packages

## Single Definition Import

Single definition import ← `import <packagename> . <definitionname>`

The *single definition import* of the form  
`import <packagename> . <definitionname>;`  
allows us to import a single specific definition (a constant or class but not a subpackage) from a package and use that definition referred to by its *definitionname* without the package prefix

```
encapsulated package ComplexUser2
import ComplexNumbers.Complex;
import ComplexNumbers.multiply;
import ComplexNumbers.add;
class User
Complex a(x=1.0, y=2.0);
Complex b(x=1.0, y=2.0);
Complex z,w;
equation
z = multiply(a,b);
w = add(a,b);
end User;
end ComplexUser2;
```

There is no risk for name collision as long as we do not try to import two definitions with the same short name

## Unqualified Import

Unqualified import ← `import <packagename> . *`

The unqualified import statement of the form  
`import packagename . *;`  
imports all definitions from the package using their short names without qualification prefixes.  
Danger: Can give rise to name collisions if imported package is changed.

```
class ComplexUser3
import ComplexNumbers.*;
Complex a(x=1.0, y=2.0);
Complex b(x=1.0, y=2.0);
Complex z,w;
equation
z = multiply(a,b);
w = add(a,b);
end ComplexUser3;
```

This example also shows direct import into a class instead of into an enclosing package

## Renaming Import

Renaming import ← `import <shortpackagename> = <packagename>`

The *renaming import* statement of the form:  
`import <shortpackagename> = <packagename>;`  
imports a package and renames it locally to *shortpackagename*.  
One can refer to imported definitions using *shortpackagename* as  
a presumably shorter package prefix.

```
class ComplexUser4
  import Co.ComplexNumbers;
  Co.Complex a(x=1.0, y=2.0);
  Co.Complex b(x=1.0, y=2.0);
  Co.Complex z, w;
  equation
    z = Co.multiply(a,b);
    w = Co.add(a,b);
  end ComplexUser4;
```

This is as safe as qualified  
import but gives more  
concise code

## Package and Library Structuring

A well-designed package structure is one of the most important aspects that influences the complexity, understandability, and maintainability of large software systems. There are many factors to consider when designing a package, e.g.:

- The name of the package.
- Structuring of the package into subpackages.
- Reusability and encapsulation of the package.
- Dependencies on other packages.

## Subpackages and Hierarchical Libraries

The main use for Modelica packages and subpackages is to structure hierarchical model libraries, of which the standard Modelica library is a good example.

```
encapsulated package Modelica           // Modelica
  encapsulated package Mechanics        // Modelica.Mechanics
    encapsulated package Rotational    // Modelica.Mechanics.Rotational
      model Inertia // Modelica.Mechanics.Rotational.Inertia
      ...
    end Inertia;
    model Torque // Modelica.Mechanics.Rotational.Torque
    ...
  end Torque;
  ...
end Rotational;
...
end Mechanics;
...
end Modelica;
```

## Encapsulated Packages and Classes

An encapsulated package or class *prevents* direct reference to public definitions *outside* itself, but as usual allows access to public subpackages and classes inside itself.

- Dependencies on other packages become explicit
  - more readable and understandable models!
- Used packages from outside must be *imported*.

```
encapsulated model TorqueUserExample1
  import Modelica.Mechanics.Rotational; // Import package Rotational
  Rotational.Torque t2; // Use Torque, OK!
  Modelica.Mechanics.Rotational.Inertia w2;
  //Error! No direct reference to the top-level Modelica package
  ... // to outside an encapsulated class
end TorqueUserExample1;
```

## within Declaration for Package Placement

Use *short names* without dots when declaring the package or class in question, e.g. on a separate file or storage unit. Use `within` to specify within which package it is to be placed.

The `within` declaration states the *prefix* needed to form the fully qualified name

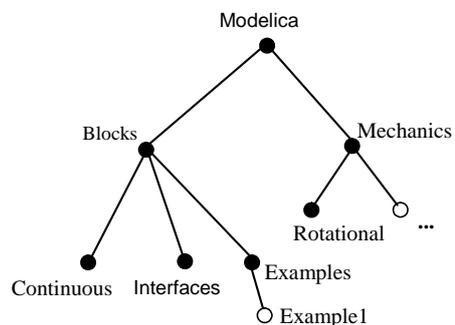
```
within Modelica.Mechanics;  
encapsulated package Rotational // Modelica.Mechanics.Rotational  
  encapsulated package Interfaces  
    import ...;  
    connector Flange_a;  
    ...  
  end Flange_a;  
  ...  
end Interfaces;  
model Inertia  
  ...  
end Inertia;  
...  
end Rotational;
```

The subpackage `Rotational` declared within `Modelica.Mechanics` has the fully qualified name `Modelica.Mechanics.Rotational`, by concatenating the *packageprefix* with the *short name* of the package.

## Mapping a Package Hierarchy into a Directory Hierarchy

A Modelica package hierarchy can be mapped into a corresponding directory hierarchy in the file system

```
C:\library  
  \Modelica  
    package.mo  
    \Blocks  
      package.mo  
      Continuous.mo  
      Interfaces.mo  
    \Examples  
      package.mo  
      Example1.mo  
    \Mechanics  
      package.mo  
      Rotational.mo  
      ...
```



## Mapping a Package Hierachy into a Directory Hirarchy

```
within;
encapsulated package Modelica
  "Modelica root package";
end Modelica;
```

It contains an empty Modelica package declaration since all subpackages under Modelica are represented as subdirectories of their own. The empty within statement can be left out if desired

```
C:\library
  \Modelica
    \package.mo
    \Blocks
      package.mo
      Continuous.mo
      Interfaces.mo
    \Examples
      package.mo
      Example1.mo
    \Mechanics
      package.mo
      Rotational.mo
      ...
```

```
within Modelica.Blocks;
encapsulated package Examples
  "Examples for Modelica.Blocks";
import ...;
end Examples;
```

```
within Modelica.Blocks.Examples;
model Example1
  "Usage example 1 for Modelica.Blocks";
  ...
end Example1;
```

```
within Modelica.Mechanics;
encapsulated package Rotational
import ...;
connector Flange_a;
...
end Flange_a;
...
end Interfaces;
model Inertia
  ...
end Inertia;
...
end Rotational;
```

The subpackage Rotational stored as the file Rotational.mo. Note that Rotational contains the subpackage Interfaces, which also is stored in the same file since we chose not to represent Rotational as a directory

# Modelica Libraries

## Modelica Standard Library

*Modelica Standard Library* (called *Modelica*) is a standardized predefined package developed by Modelica Association

It can be used freely for both commercial and noncommercial purposes under the conditions of *The Modelica License*.

Modelica libraries are available online including documentation and source code from <http://www.modelica.org/library/library.html>.

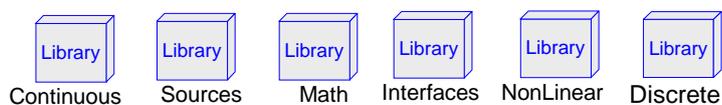
## Modelica Standard Library cont'

Modelica Standard Library contains components from various application areas, with the following sublibraries:

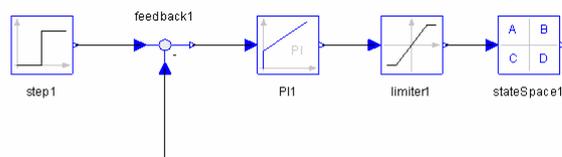
- Blocks Library for basic input/output control blocks
- Constants Mathematical constants and constants of nature
- Electrical Library for electrical models
- Icons Icon definitions
- Math Mathematical functions
- Mechanics Library for mechanical systems
- Media Media models for liquids and gases
- Slunits Type definitions based on SI units according to ISO 31-1992
- Stategraph Hierarchical state machines (analogous to Statecharts)
- Thermal Components for thermal systems
- Utility Utilities Utility functions especially for scripting

## Modelica.Blocks

This library contains input/output blocks to build up block diagrams.



Example:



## Modelica.Constants

A package with often needed constants from mathematics, machine dependent constants, and constants of nature.

Examples:

```
constant Real pi=2*Modelica.Math.asin(1.0);
```

```
constant Real small=1.e-60 "Smallest number such that small and -small  
are representable on the machine";
```

```
constant Real G(final unit="m3/(kg.s2)") = 6.673e-11 "Newtonian constant  
of gravitation";
```

```
constant Real h(final unit="J.s") = 6.62606876e-34 "Planck constant";
```

```
constant Modelica.SIunits.CelsiusTemperature T_zero=-273.15 "Absolute  
zero temperature";
```

## Modelica.Electrical

Electrical components for building analog, digital, and multiphase circuits



Analog



Digital

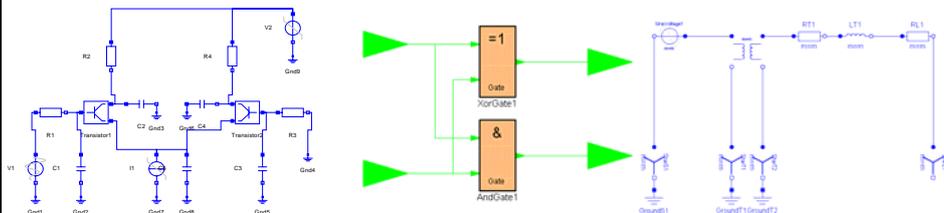


Machines



MultiPhase

Examples:



## Modelica.Icons

Package with icons that can be reused in other libraries

Examples:



Info



Library1



Library2



Example



RotationalSensor



TranslationalSensor



GearIcon



MotorIcon

## Modelica.Math

Package containing basic mathematical functions:

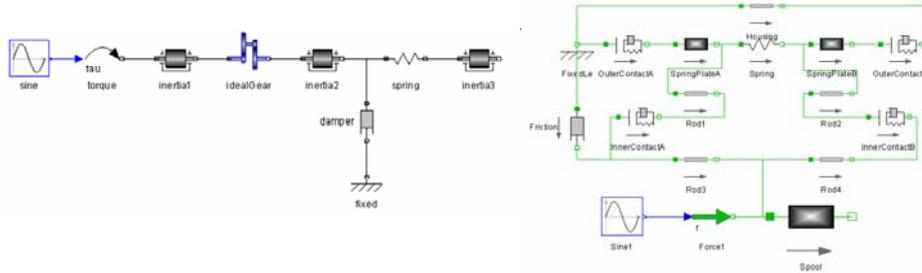
$\sin(u)$	sine
$\cos(u)$	cosine
$\tan(u)$	tangent ( $u$ shall not be: $\dots, -\pi/2, \pi/2, 3\pi/2, \dots$ )
$\text{asin}(u)$	inverse sine ( $-1 \leq u \leq 1$ )
$\text{acos}(u)$	inverse cosine ( $-1 \leq u \leq 1$ )
$\text{atan}(u)$	inverse tangent
$\text{atan2}(u1, u2)$	four quadrant inverse tangent
$\sinh(u)$	hyperbolic sine
$\cosh(u)$	hyperbolic cosine
$\tanh(u)$	hyperbolic tangent
$\exp(u)$	exponential, base e
$\log(u)$	natural (base e) logarithm ( $u > 0$ )
$\log10(u)$	base 10 logarithm ( $u > 0$ )

## Modelica.Mechanics

Package containing components for mechanical systems

Subpackages:

- Rotational 1-dimensional rotational mechanical components
- Translational 1-dimensional translational mechanical components
- MultiBody 3-dimensional mechanical components



## Modelica.SIunits

This package contains predefined types based on the international standard of units:

- ISO 31-1992 “General principles concerning quantities, units and symbols”
- ISO 1000-1992 “SI units and recommendations for the use of their multiples and of certain other units”.

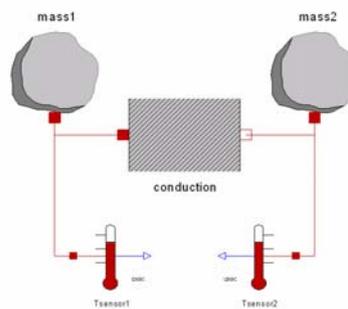
A subpackage called `NonSIunits` is available containing non SI units such as `Pressure_bar`, `Angle_deg`, etc

## Modelica.Thermal

Subpackage `FluidHeatFlow` with components for heat flow modeling.

Sub package `HeatTransfer` with components to model 1-dimensional heat transfer with lumped elements

Example:



## ModelicaAdditions Library (OLD)

`ModelicaAdditions` library contains additional Modelica libraries from DLR. This has been largely replaced by the new release of the Modelica 2.1 libraries.

Sublibraries:

- `Blocks` Input/output block sublibrary
- `HeatFlow1D` 1-dimensional heat flow (replaced by `Modelica.Thermal`)
- `Multibody` Modelica library to model 3D mechanical systems
- `PetriNets` Library to model Petri nets and state transition diagrams
- `Tables` Components to interpolate linearly in tables

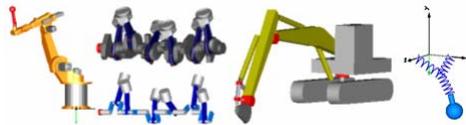
## ModelicaAdditions.Multibody (OLD)

This is a Modelica library to model 3D Mechanical systems including visualization

New version has been released (march 2004) that is called Modelica.Mechanics.MultiBody in the standard library

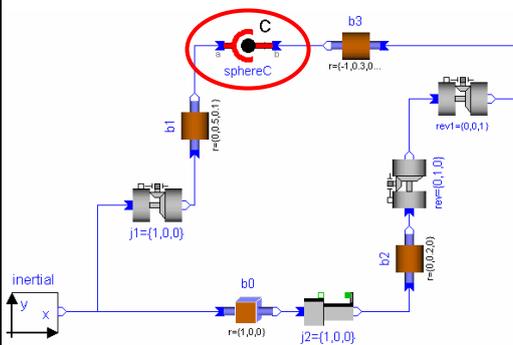
Improvements:

- Easier to use
- Automatic handling of kinematic loops.
- Built-in animation properties for all components

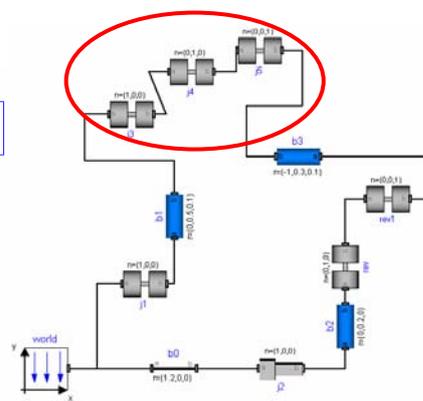


## MultiBody (MBS) - Example Kinematic Loop

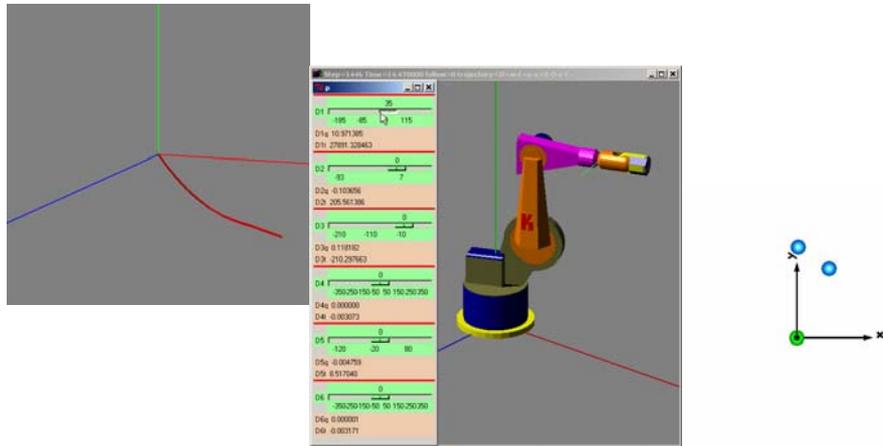
Old library  
(cutjoint needed)



New library  
(no cutjoint needed)



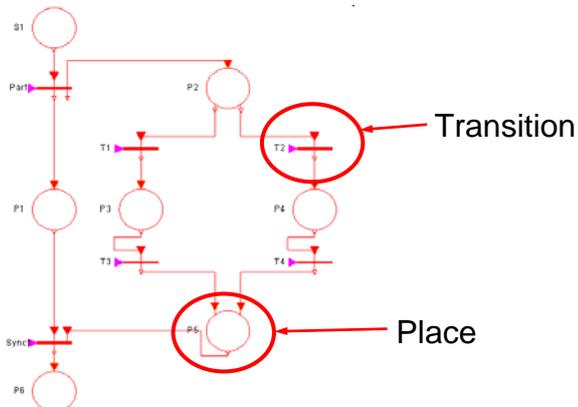
## MultiBody (MBS) - Example Animations



## ModelicaAdditions.PetriNets

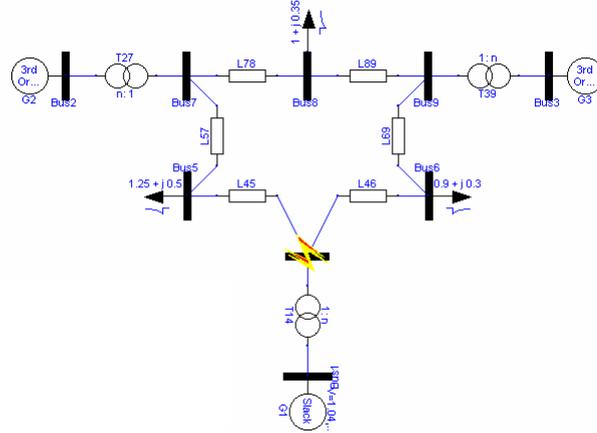
This package contains components to model Petri nets

Used for modeling of computer hardware, software, assembly lines, etc



## Power System Stability - ObjectStab

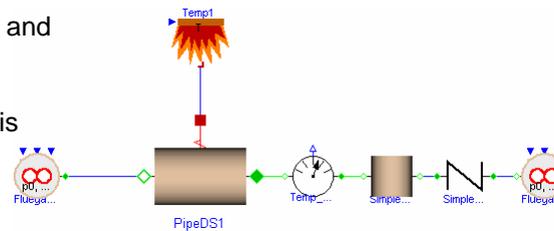
The ObjectStab package is a Modelica Library for Power Systems Voltage and Transient stability simulations



## Thermo-hydraulics Library - ThermoFluid

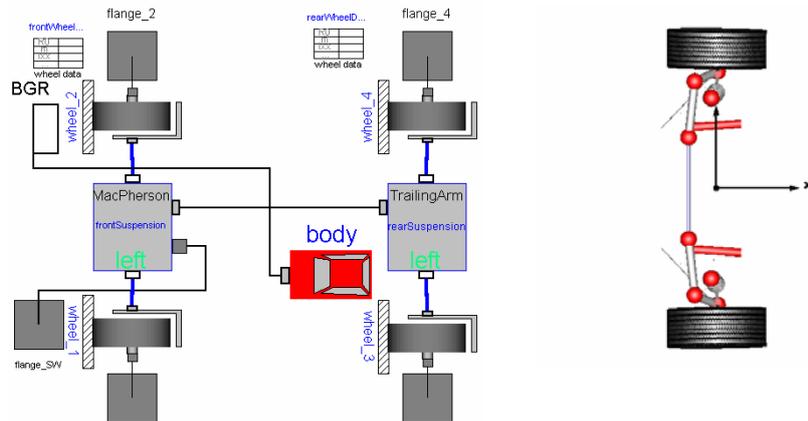
ThermoFluid is a Modelica base library for thermo-hydraulic models

- Includes models that describe the basic physics of flows of fluid and heat, medium property models for water, gases and some refrigerants, and also simple components for system modeling.
- Handles static and dynamic momentum balances
- Robust against backwards and zero flow
- The discretization method is a first-order, finite volume method (staggered grid).



## Vehicle Dynamics Library - VehicleDynamics

This library is used to model vehicle chassis



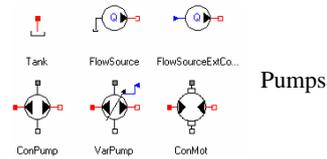
## Some Other Free Libraries

- ExtendedPetriNets      Petri nets and state transition diagrams (extended version)
- QSSFluidFlow            Quasi Steady-State Fluid Flows
- SystemDynamics        System Dynamics Formalism
- Atplus                     Building Simulation and Building Control (includes Fuzzy Control library)
- ThermoPower            Thermal power plants
- WasteWater              Library for biological wastewater treatment plants
- SPICELib                 Support modeling and analysis capabilities of the circuit simulator PSPICE

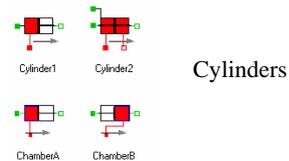
*Read more about the libraries at [www.modelica.org/library/library.html](http://www.modelica.org/library/library.html)*

## Hydraulics Library HyLib

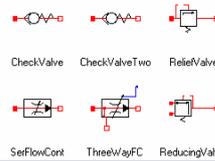
- Licensed Modelica package developed by Peter Beater
- More than 90 models for
  - Pumps
  - Motors and cylinders
  - Restrictions and valves
  - Hydraulic lines
  - Lumped volumes and sensors
- Models can be connected in an arbitrary way, e.g. in series or in parallel.
- HyLibLight is a free subset of HyLib
- More info: [www.hylib.com](http://www.hylib.com)



Pumps



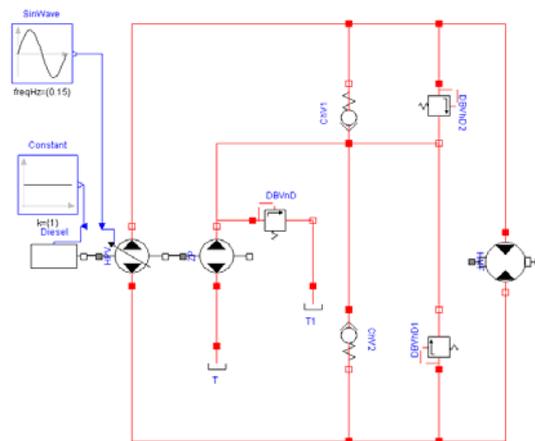
Cylinders



Valves

## HyLib - Example

Hydraulic drive system with closed circuit



## Pneumatics Library PneuLib

- Licensed Modelica package developed by Peter Beater

- More than 80 models for

- Cylinders
- Motors
- Valves and nozzles
- Lumped volumes
- Lines and sensors

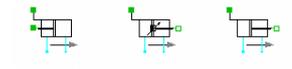
- Models can be connected in an arbitrary way, e.g. in series or in parallel.

- PneuLibLight is a free subset of HyLib.

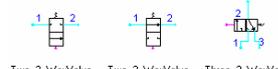
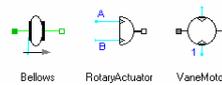
- More info: [www.pneulib.com](http://www.pneulib.com)



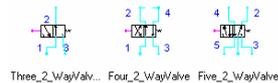
Directional valves



Flow control valves

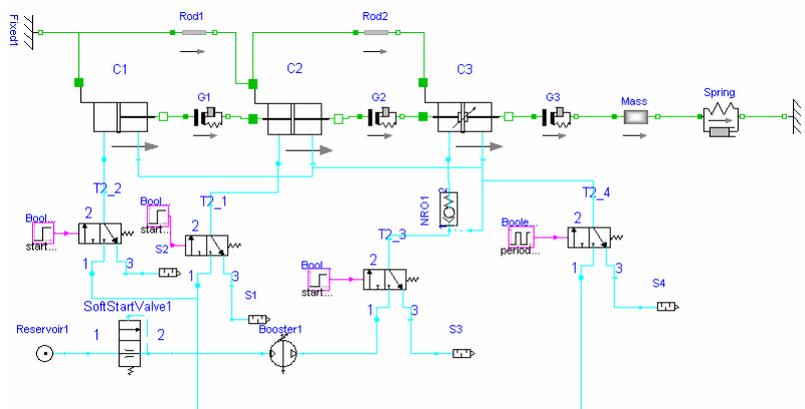


Cylinders



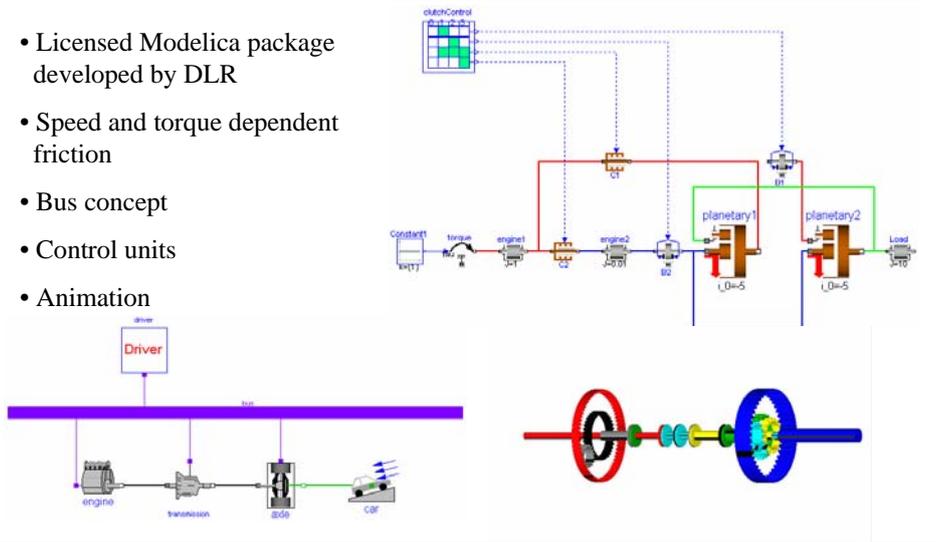
## PneuLib - Example

Pneumatic circuit with multi-position cylinder, booster and different valves



## Powertrain Library - Powertrain

- Licensed Modelica package developed by DLR
- Speed and torque dependent friction
- Bus concept
- Control units
- Animation

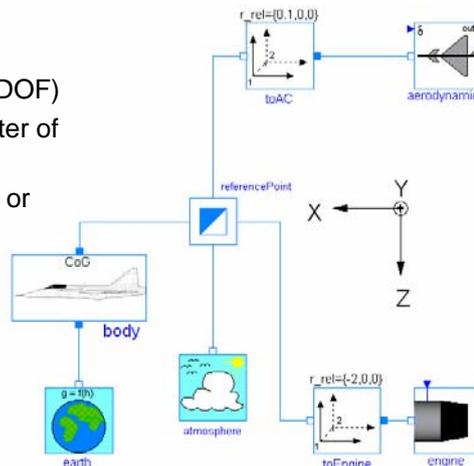


## Some Modelica Applications

## Example Fighter Aircraft Library

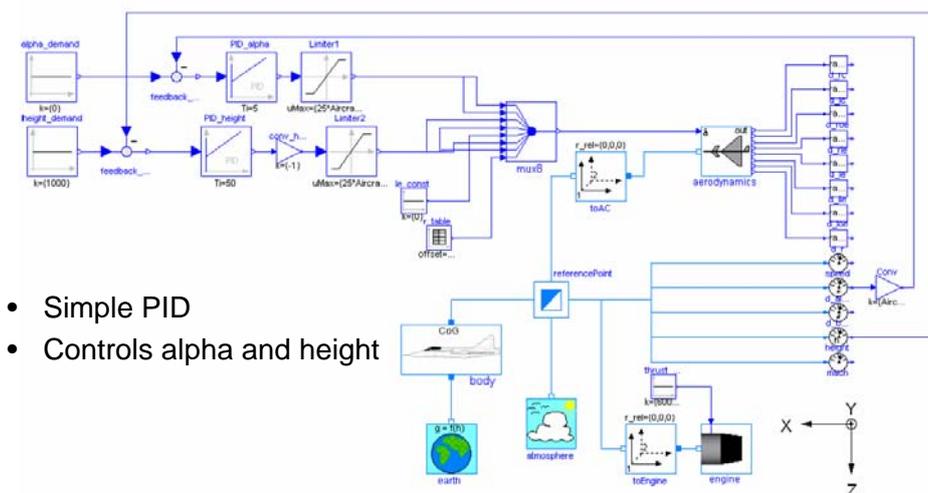
Custom made library, Aircraft\*, for fighter aircraft applications

- Six degrees of freedom (6 DOF)
- Dynamic calculation of center of gravity (CoG)
- Use of Aerodynamic tables or mechanical rudders



\*Property of FOI (The Swedish Defence Institute)

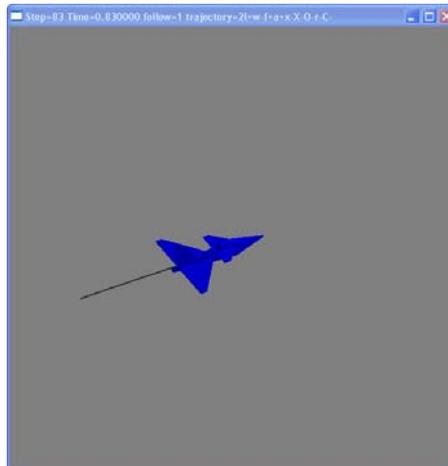
## Aircraft with Controller



- Simple PID
- Controls alpha and height

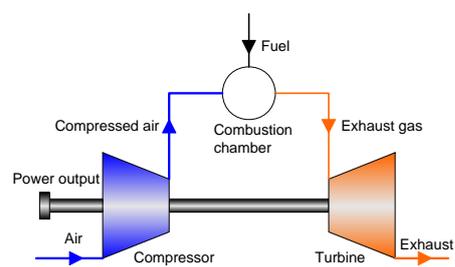
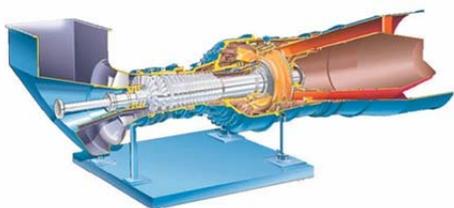
## Example Aircraft Animation

Animation of fighter aircraft with controller

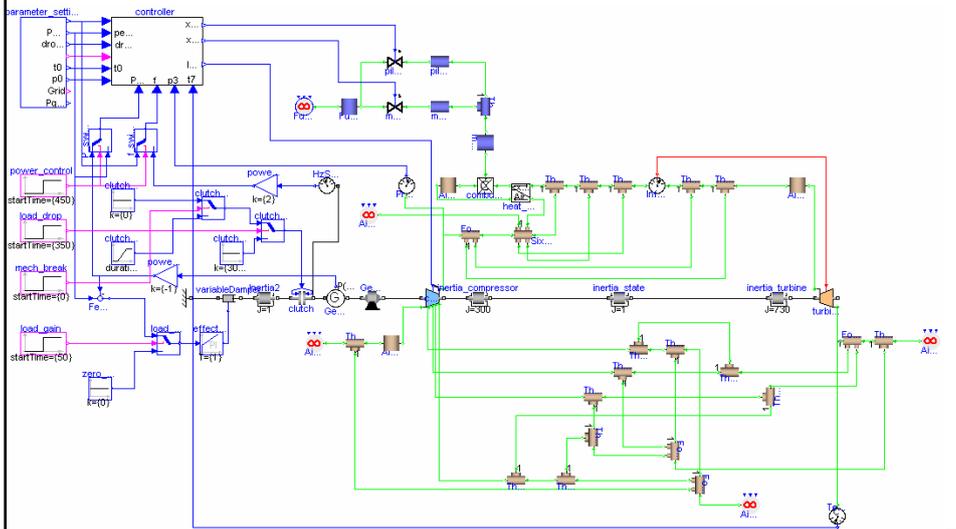


## Example Gas Turbine

42 MW gas turbine (GTX 100) from Siemens Industrial Turbomachinery AB, Finspång, Sweden



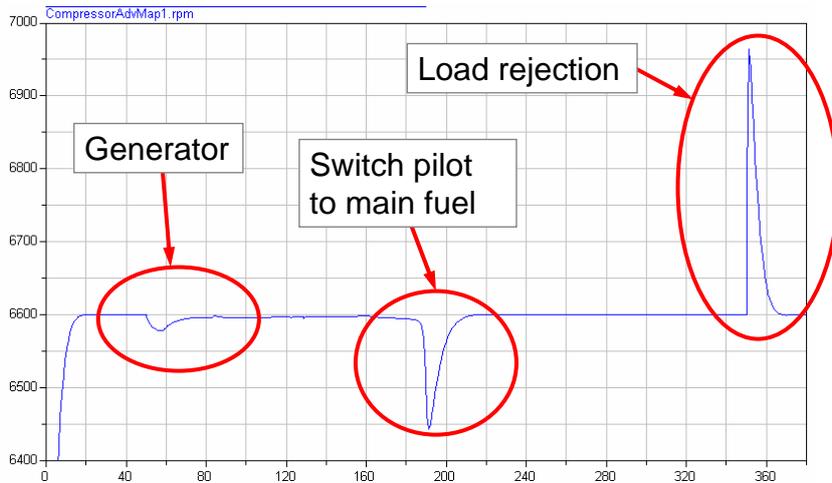
## Example Gas Turbine



31 Peter Fritzon

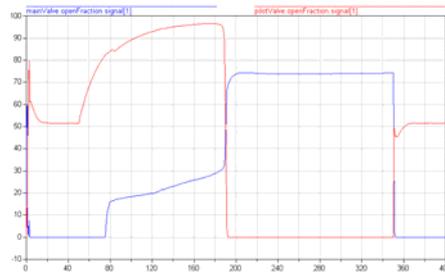
## Example Gas Turbine – Load Rejection

Rotational speed (rpm) of the compressor shaft

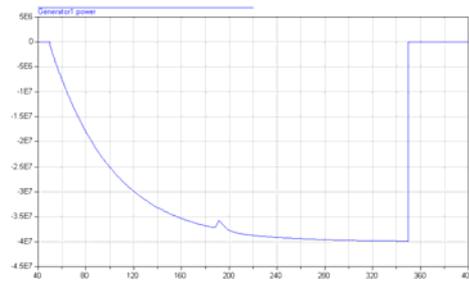


32 Peter Fritzon

## Example Gas Turbine – Load Rejection



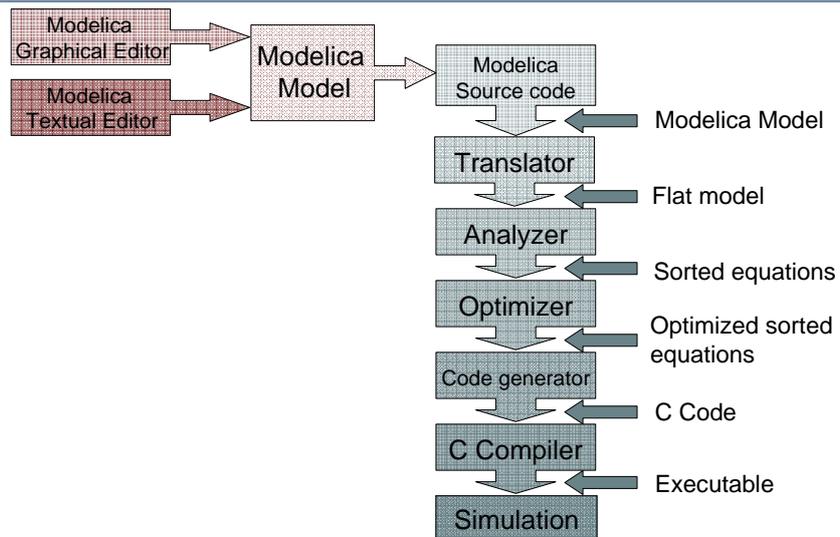
Percentage of fuel valve opening  
(red = pilot, blue = main)



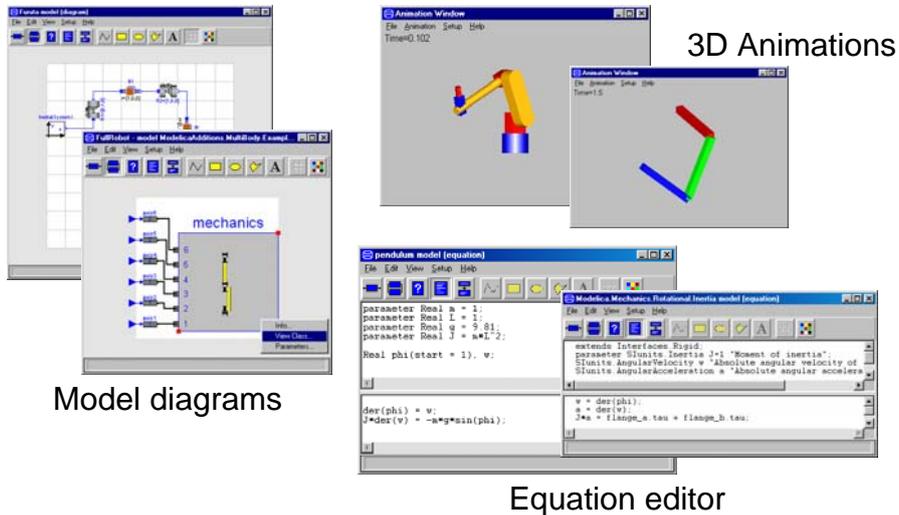
Generated power to the simulated  
electrical grid

# Modeling and Simulation Environments

## The Translation Process



## Commercial Environments – Dymola from Dynasim



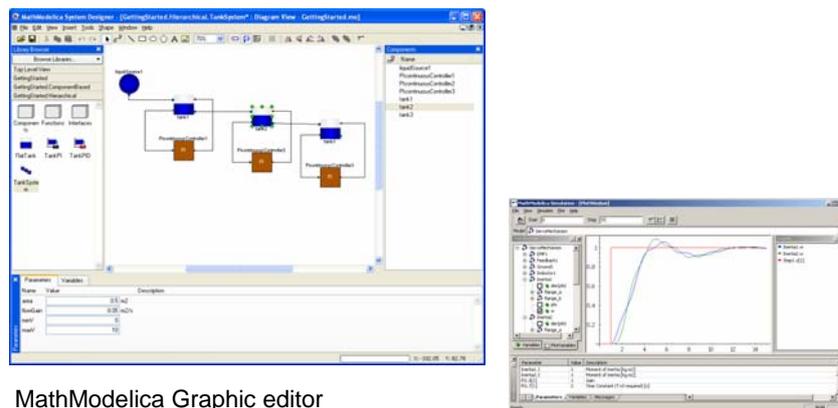
Model diagrams

3D Animations

Equation editor

Courtesy of Dynasim AB, Sweden

## Commercial Environments – MathModelica System Designer from MathCore



MathModelica Graphic editor

Simulation Center

Courtesy of Mathcore Engineering AB

## OpenModelica Environment

The goal of the OpenModelica project is to:

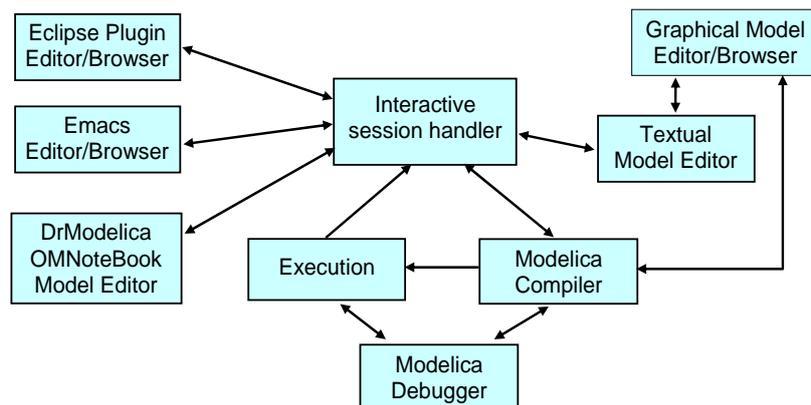
- Create a **complete** Modelica modeling, compilation and simulation environment.
- Provide **free** software distributed in **binary** and **source code** form.
- Provide a modeling and simulation environment for **research** and industrial purposes.
- Develop a formal semantics of Modelica

Features of currently available implementation:

- Command shell environment allows to enter and evaluate Modelica declarations, expressions, assignments, and function calls.
- Modelica functions are implemented, including array support.
- Modelica equations are implemented, but with certain limitations.
- Packages, inheritance, modifiers, etc. are implemented.
- etc.

<http://www.ida.liu.se/~pelab/modelica/OpenModelica.html>

## OpenModelica Environment Architecture



<http://www.ida.liu.se/projects/OpenModelica>

# OMNotebook Electronic Book with Modelica Exercises and OMShell Interactive Shell

**First Basic Class**

**1 HelloWorld**

The program contains a declaration of a class called HelloWorld with two fields and one equation. The first field is the variable  $x$  which is initialized to a start value 1 at the time when the simulation starts. The second field is the variable  $a$  which is a constant that is initialized to 2 at the beginning of the simulation. Such a constant is prefixed by the keyword parameter in order to indicate that it is constant during simulation but is a model-parameter that can be changed between simulations.

The Modelica program solves a second differential equation:  $\ddot{x} = -a \cdot x$ . The variable  $x$  is a state variable that can change value over time. The  $\dot{x}$  is the time derivative of  $x$ .

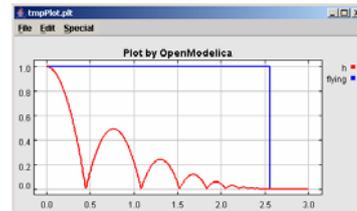
```
class HelloWorld
  Real x(start = 1);
  parameter Real a = 2;
  equation
    der(x) = - a * x;
end HelloWorld;
ok.
```

**2 Simulation of HelloWorld**

```
simulate(HelloWorld, startTime=0, stopTime=4);
[done]
```

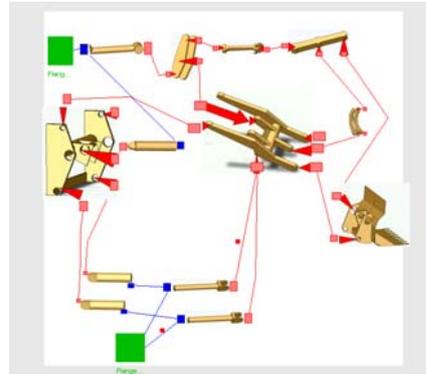
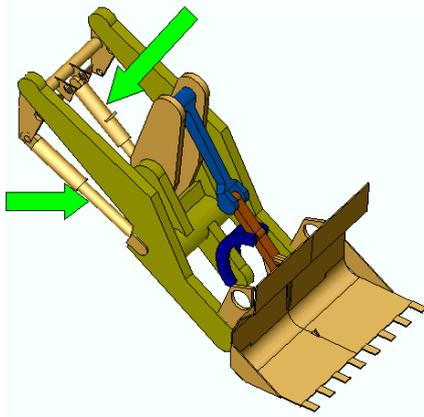
```
plot(x);
```

```
>>simulate(BouncingBall, stopTime=3.0);
>>plot({h, flying});
```

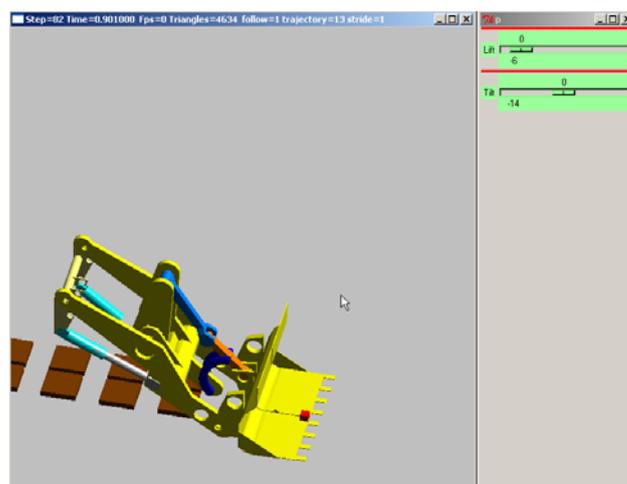


# Examples of Applications (usually using commercial tools)

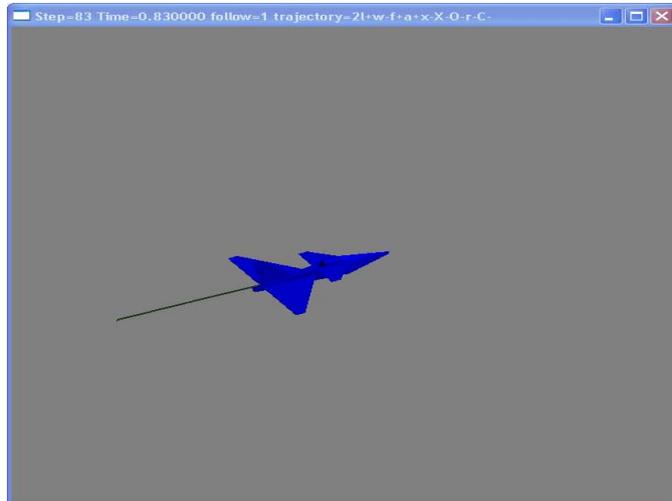
## Example - Modeling of a Wheel Loader Lifter



## Simulation of a Wheel Loader Lifter



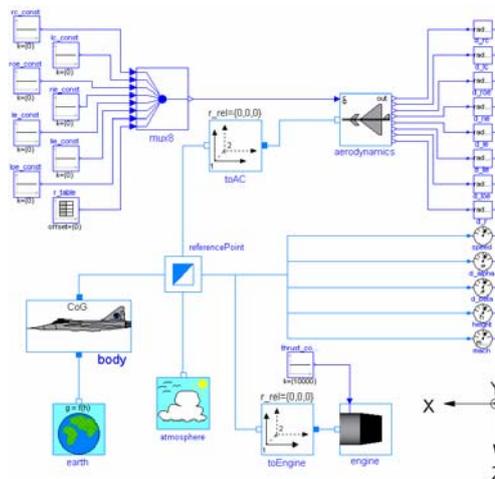
# Modelica Simulation of AirCRAFT Dynamics



Developed by MathCore  
for the Swedish Defense  
Research Institute (FOI)

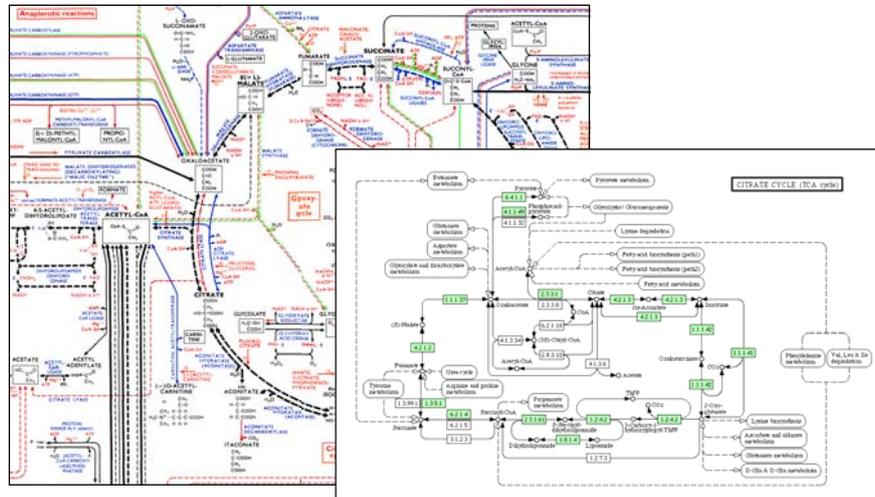
# Modelica AirCRAFT Component Library

Model Structure – Using a Modelica AirCRAFT Component  
Library developed by MathCore  
for the Swedish Defense  
Research Institute (FOI)



Courtesy of Swedish Defense Research Institute (FOI)

## Pathways in a Biochemical System



## Examples of Modelica Research

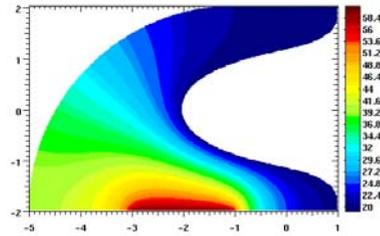
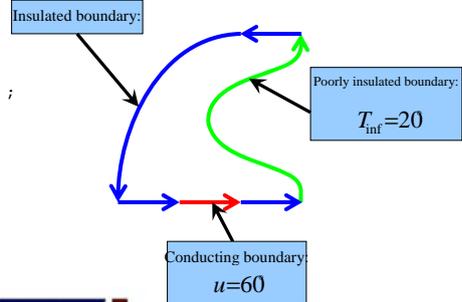
- PDEs in Modelica
- Debugging
- Parallelization
- Language Design for Meta Programming
- Variant Handling
- Biochemical modeling

## Extending Modelica with PDEs for 2D, 3D flow problems

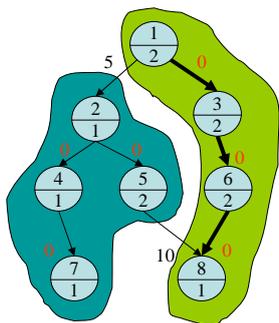
```

class PDEModel
  HeatNeumann h_iso;
  Dirichlet h_heated(g=50);
  HeatRobin h_glass(h_heat=30000);
  HeatTransfer ht;
  Rectangle2D dom;
equation
  dom.eq = ht;
  dom.left.bc = h_glass;
  dom.top.bc = h_iso;
  dom.right.bc = h_iso;
  dom.bottom.bc = h_heated;
end PDEModel;

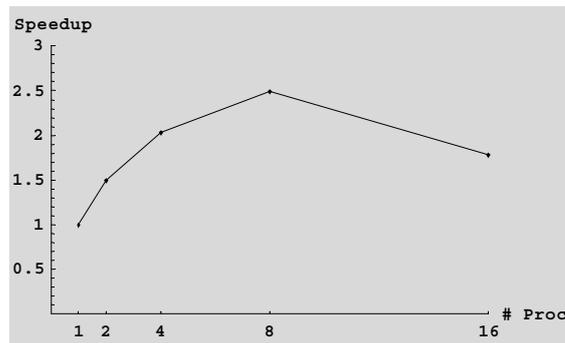
```



## Automatic Generation of Parallel Code from Modelica Equation-Based Models

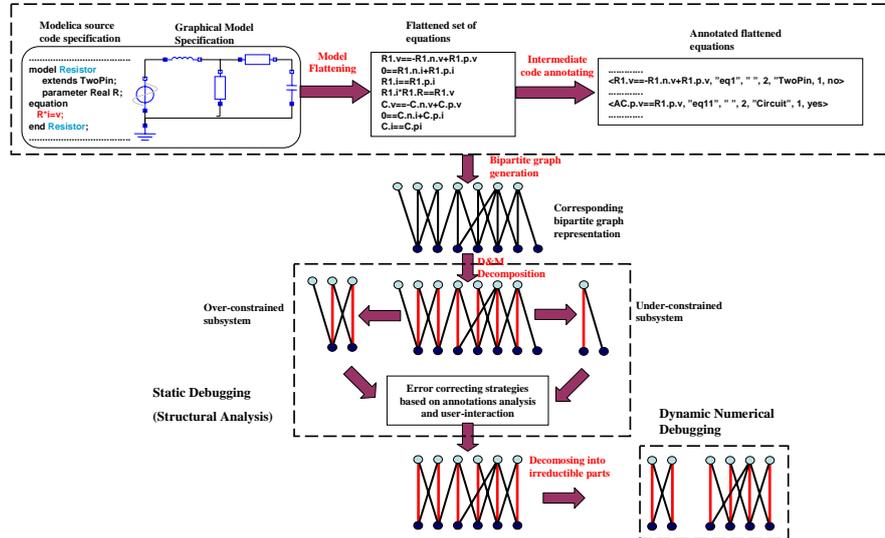


Clustered Task Graph



Thermofluid Pipe Application

## Equation Debugger General Architecture



## Conclusions

Modelica has a good chance to become the next generation computational modeling language

Two complete commercial Modelica implementations currently available (MathModelica, Dymola), and an open source implementation (OpenModelica) under development

## Contact

[www.ida.liu.se/projects/OpenModelica](http://www.ida.liu.se/projects/OpenModelica)

Download OpenModelica and drModelica, book chapter

[www.mathcore.com](http://www.mathcore.com)

MathModelica Tool

[www.mathcore.com/drModelica](http://www.mathcore.com/drModelica)

Book web page, Download book chapter

[www.modelica.org](http://www.modelica.org)

Modelica Association

[petfr@ida.liu.se](mailto:petfr@ida.liu.se)

[OpenModelica@ida.liu.se](mailto:OpenModelica@ida.liu.se)

# Biological Models

## Population Dynamics

### Predator-Prey

## Some Well-known Population Dynamics Applications

- Population Dynamics of Single Population
- Predator-Prey Models (e.g. Foxes and Rabbits)

## Population Dynamics of Single Population

- $P$  – population size = number of individuals in a population
- $\dot{P}$  – population change rate, change per time unit
- $g$  – growth factor of population (e.g. % births per year)
- $d$  – death factor of population (e.g. % deaths per year)

$$\text{growthrate} = g \cdot P$$

$$\text{deathrate} = d \cdot P$$

*Exponentially increasing  
population if  $(g-d) > 0$*

$$\dot{P} = \text{growthrate} - \text{deathrate}$$

*Exponentially decreasing  
population if  $(g-d) < 0$*

$$\dot{P} = (g - d) \cdot P$$

## Population Dynamics Model

- $g$  – growth rate of population
- $d$  – death rate of population
- $P$  – population size

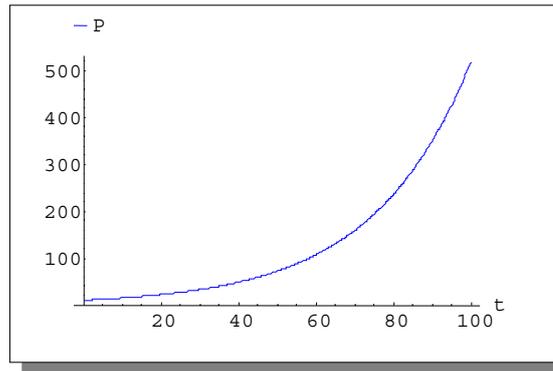
$$\dot{P} = \text{growthrate} - \text{deathrate}$$

```
class PopulationGrowth
  parameter Real g = 0.04 "Growth factor of population";
  parameter Real d = 0.0005 "Death factor of population";
  Real P(start=10) "Population size, initially 10";
equation
  der(P) = (g-d)*P;
end PopulationGrowth;
```

## Simulation of PopulationGrowth

```
simulate(PopulationGrowth, stopTime=100)  
plot(P)
```

*Exponentially increasing  
population if  $(g-d)>0$*



## Population Growth Exercise!!

- Locate the PopulationGrowth model in DrModelica
- Change the initial population size and growth and death factors to get an exponentially decreasing population

```
simulate(PopulationGrowth, stopTime=100)  
plot(P)
```

*Exponentially decreasing  
population if  $(g-d)<0$*

```
class PopulationGrowth  
  parameter Real g = 0.04 "Growth factor of population";  
  parameter Real d = 0.0005 "Death factor of population";  
  Real P(start=10) "Population size, initially 10";  
equation  
  der(P) = (g-d)*P;  
end PopulationGrowth;
```

## Population Dynamics with both Predators and Prey Populations

- Predator-Prey models

## Predator-Prey (Foxes and Rabbits) Model

- $R$  = rabbits = size of rabbit population
- $F$  = foxes = size of fox population
- $\dot{R}$  = der(rabbits) = change rate of rabbit population
- $\dot{F}$  = der(foxes) = change rate of fox population
- $g_r = g_r$  = growth factor of rabbits
- $d_f = d_f$  = death factor of foxes
- $d_{rf} = d_{rf}$  = death factor of rabbits due to foxes
- $g_{fr} = g_{fr}$  = growth factor of foxes due to rabbits and foxes

$$\dot{R} = g_r \cdot R - d_{rf} \cdot F \cdot R \quad \dot{F} = g_{fr} \cdot d_{rf} \cdot R \cdot F - d_f \cdot F$$

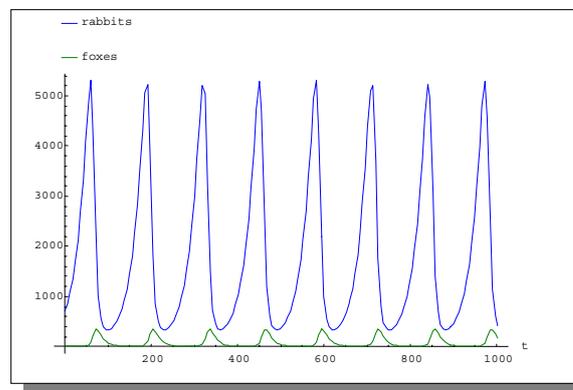
```
der(rabbits) = g_r*rabbits - d_rf*rabbits*foxes;  
der(foxes)   = g_fr*d_rf*rabbits*foxes - d_f*foxes;
```

## Predator-Prey (Foxes and Rabbits) Model

```
class LotkaVolterra
  parameter Real g_r =0.04    "Natural growth rate for rabbits";
  parameter Real d_rf=0.0005  "Death rate of rabbits due to foxes";
  parameter Real d_f =0.09    "Natural deathrate for foxes";
  parameter Real g_fr=0.1    "Efficiency in growing foxes from rabbits";
  Real rabbits(start=700)    "Rabbits, (R) with start population 700";
  Real foxes(start=10)      "Foxes, (F) with start population 10";
equation
  der(rabbits) = g_r*rabbits - d_rf*rabbits*foxes;
  der(foxes)   = g_fr*d_rf*rabbits*foxes - d_f*foxes;
end LotkaVolterra;
```

## Simulation of Predator-Prey (LotkaVolterra)

```
simulate(LotkaVolterra, stopTime=3000)
plot({rabbits, foxes}, xrange={0,1000})
```



## Exercise of Predator-Prey

- Locate the LotkaVolterra model in DrModelica
- Change the death and growth rates for foxes and rabbits, simulate, and observe the effects

```
simulate(LotkaVolterra, stopTime=3000)  
plot({rabbits, foxes}, xrange={0,1000})
```

```
class LotkaVolterra  
  parameter Real g_r =0.04 "Natural growth rate for rabbits";  
  parameter Real d_rf=0.0005 "Death rate of rabbits due to foxes";  
  parameter Real d_f =0.09 "Natural deathrate for foxes";  
  parameter Real g_fr=0.1 "Efficency in growing foxes from rabbits";  
  Real rabbits(start=700) "Rabbits, (R) with start population 700";  
  Real foxes(start=10) "Foxes, (F) with start population 10";  
equation  
  der(rabbits) = g_r*rabbits - d_rf*rabbits*foxes;  
  der(foxes) = g_fr*d_rf*rabbits*foxes - d_f*foxes;  
end LotkaVolterra;
```

# Model Design

## Modeling Approaches

- Traditional state space approach
- Traditional signal-style block-oriented approach
- Object-oriented approach based on finished library component models
- Object-oriented flat model approach
- Object-oriented approach with design of library model components

## Modeling Approach 1

Traditional state space approach

## Traditional State Space Approach

- Basic structuring in terms of subsystems and variables
- Stating equations and formulas
- Converting the model to state space form:

$$\dot{x}(t) = f(x(t), u(t))$$

$$y(t) = g(x(t), u(t))$$

## Difficulties in State Space Approach

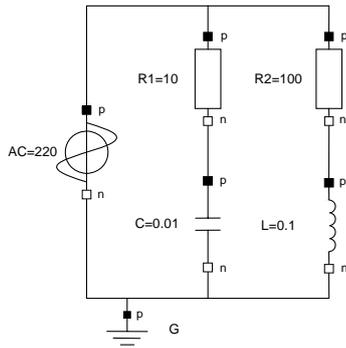
- The system decomposition does not correspond to the "natural" physical system structure
- Breaking down into subsystems is difficult if the connections are not of input/output type.
- Two connected state-space subsystems do not usually give a state-space system automatically.

## Modeling Approach 2

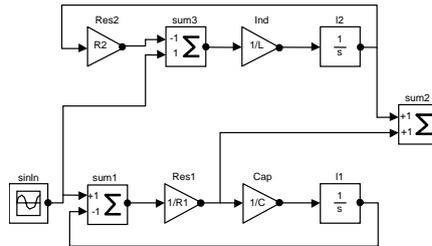
Traditional signal-style block-oriented approach

## Physical Modeling Style (e.g. Modelica) vs signal flow Block-Oriented Style (e.g. Simulink)

Modelica:  
Physical model – easy to understand



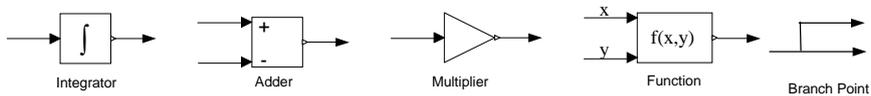
Block-oriented:  
Signal-flow model – hard to understand for physical systems



## Traditional Block Diagram Modeling

- Special case of model components: the causality of each interface variable has been fixed to either *input* or *output*

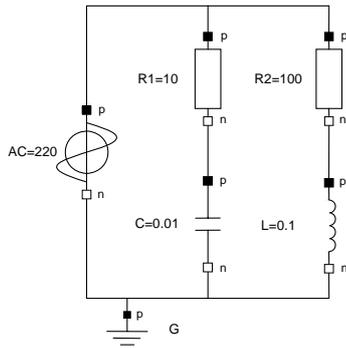
Typical Block diagram model components:



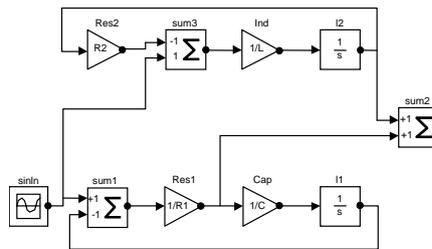
Simulink is a common block diagram tool

## Physical Modeling Style (e.g Modelica) vs signal flow Block-Oriented Style (e.g. Simulink)

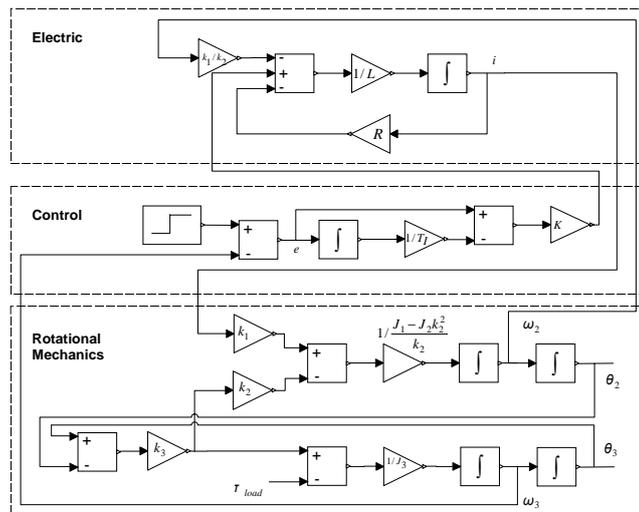
Modelica:  
Physical model – easy to understand



Block-oriented:  
Signal-flow model – hard to understand for physical systems



## Example Block Diagram Models



## Properties of Block Diagram Modeling

- - The system decomposition topology does not correspond to the "natural" physical system structure
- - Hard work of manual conversion of equations into signal-flow representation
- - Physical models become hard to understand in signal representation
- - Small model changes (e.g. compute positions from force instead of force from positions) requires redesign of whole model
- + Block diagram modeling works well for control systems since they are signal-oriented rather than "physical"

## Object-Oriented Modeling Variants

- Approach 3: Object-oriented approach based on finished library component models
- Approach 4: Object-oriented flat model approach
- Approach 5: Object-oriented approach with design of library model components

## Object-Oriented Component-Based Approaches in General

- Define the system briefly
  - What kind of system is it?
  - What does it do?
- Decompose the system into its most important components
  - Define communication, i.e., determine interactions
  - Define interfaces, i.e., determine the external ports/connectors
  - Recursively decompose model components of “high complexity”
- Formulate new model classes when needed
  - Declare new model classes.
  - Declare possible base classes for increased reuse and maintainability

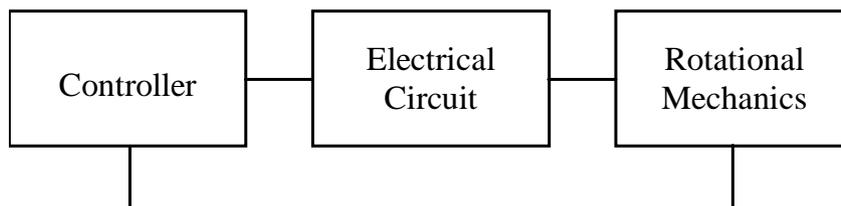
## Top-Down versus Bottom-up Modeling

- Top Down: Start designing the overall view. Determine what components are needed.
- Bottom-Up: Start designing the components and try to fit them together later.

## Approach 3: Top-Down Object-oriented approach using library model components

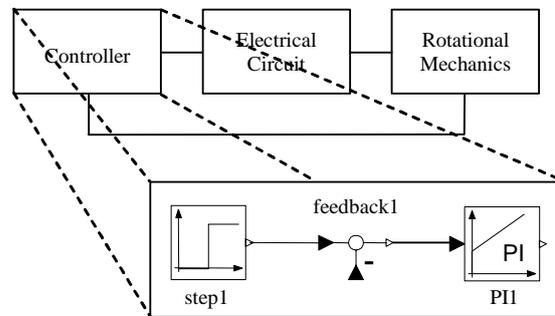
- Decompose into subsystems
- Sketch communication
- Design subsystems models by connecting library component models
- Simulate!

## Decompose into Subsystems and Sketch Communication – DC-Motor Servo Example



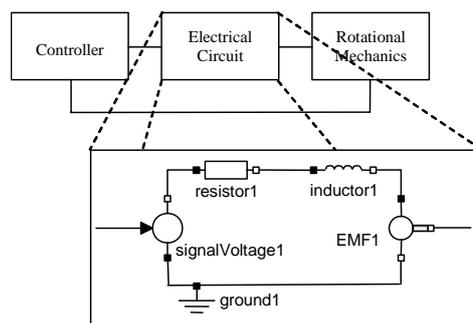
The DC-Motor servo subsystems and their connections

## Modeling the Controller Subsystem



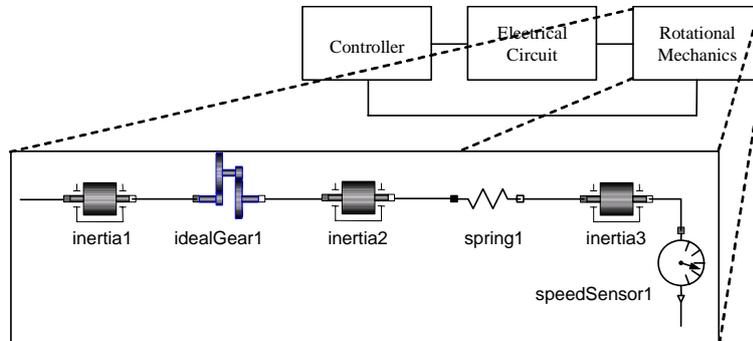
Modeling the controller

## Modeling the Electrical Subsystem



Modeling the electric circuit

## Modeling the Mechanical Subsystem

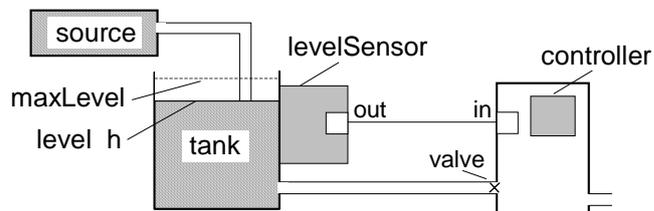


Modeling the mechanical subsystem including the speed sensor.

## Object-Oriented Modeling from Scratch

- Approach 4: Object-oriented flat model approach
- Approach 5: Object-oriented approach with design of library model components

## Example: OO Modeling of a Tank System



- The system is naturally decomposed into components

## Object-Oriented Modeling

Approach 4: Object-oriented flat model design

## Tank System Model FlatTank – No Graphical Structure

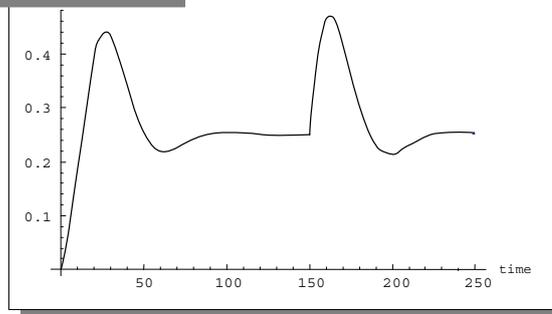
- No component structure
- Just flat set of equations
- Straight-forward but less flexible, no graphical structure

```
model FlatTank
// Tank related variables and parameters
parameter Real flowLevel(unit="m3/s")=0.02;
parameter Real area(unit="m2") =1;
parameter Real flowGain(unit="m2/s") =0.05;
Real h(start=0,unit="m") "Tank level";
Real qInflow(unit="m3/s") "Flow through input valve";
Real qOutflow(unit="m3/s") "Flow through output valve";
// Controller related variables and parameters
parameter Real K=2 "Gain";
parameter Real T(unit="s")= 10 "Time constant";
parameter Real minV=0, maxV=10; // Limits for flow output
Real ref = 0.25 "Reference level for control";
Real error "Deviation from reference level";
Real outCtr "Control signal without limiter";
Real x; "State variable for controller";
equation
assert(minV>=0,"minV must be greater or equal to zero");//
der(h) = (qInflow-qOutflow)/area; // Mass balance equation
qInflow = if time>150 then 3*flowLevel else flowLevel;
qOutflow = LimitValue(minV,maxV,-flowGain*outCtr);
error = ref-h;
der(x) = error/T;
outCtr = K*(error+x);
end FlatTank;
```

## Simulation of FlatTank System

- Flow increase to flowLevel at time 0
- Flow increase to 3\*flowLevel at time 150

```
simulate(FlatTank, stopTime=250)
plot(h, stopTime=250)
```

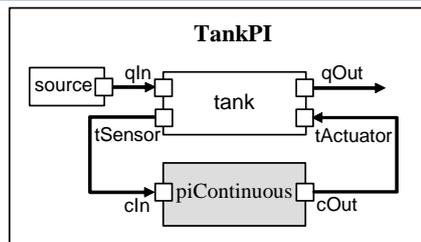


## Object-Oriented Modeling

- Approach 5:  
Object-oriented approach with design of library model components

## Object Oriented Component-Based Approach Tank System with Three Components

- Liquid source
- Continuous PI controller
- Tank



```
model TankPI
  LiquidSource          source (flowLevel=0.02);
  PIcontinuousController piContinuous (ref=0.25);
  Tank                  tank (area=1);
equation
  connect (source.qOut, tank.qIn);
  connect (tank.tActuator, piContinuous.cOut);
  connect (tank.tSensor, piContinuous.cIn);
end TankPI;
```

## Tank model

- The central equation regulating the behavior of the tank is the mass balance equation (input flow, output flow), assuming constant pressure

```

model Tank
  ReadSignal tSensor "Connector, sensor reading tank level (m)";
  ActSignal tActuator "Connector, actuator controlling input flow";
  LiquidFlow qIn "Connector, flow (m3/s) through input valve";
  LiquidFlow qOut "Connector, flow (m3/s) through output valve";
  parameter Real area(unit="m2") = 0.5;
  parameter Real flowGain(unit="m2/s") = 0.05;
  parameter Real minV=0, maxV=10; // Limits for output valve flow
  Real h(start=0.0, unit="m") "Tank level";
equation
  assert(minV>=0,"minV - minimum Valve level must be >= 0");//
  der(h) = (qIn.lflow-qOut.lflow)/area; // Mass balance
equation
  qOut.lflow = LimitValue(minV,maxV,-flowGain*tActuator.act);
  tSensor.val = h;
end Tank;
  
```

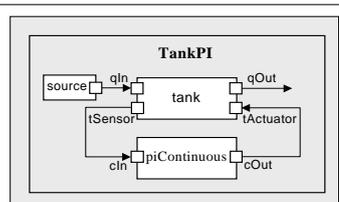
## Connector Classes and Liquid Source Model for Tank System

```

connector ReadSignal "Reading fluid level"
  Real val(unit="m");
end ReadSignal;

connector ActSignal "Signal to actuator
for setting valve position"
  Real act;
end ActSignal;

connector LiquidFlow "Liquid flow at inlets or outlets"
  Real lflow(unit="m3/s");
end LiquidFlow;
  
```



```

model LiquidSource
  LiquidFlow qOut;
  parameter flowLevel = 0.02;
equation
  qOut.lflow = if time>150 then 3*flowLevel else flowLevel;
end LiquidSource;
  
```

## Continuous PI Controller for Tank System

- error = (reference level – actual tank level)
- T is a time constant
- x is controller state variable
- K is a gain factor

$$\frac{dx}{dt} = \frac{error}{T}$$

$$outCtr = K * (error + x)$$

Integrating equations gives  
Proportional & Integrative (PI)

$$outCtr = K * (error + \int \frac{error}{T} dt)$$

```

model PIcontinuousController
  extends BaseController(K=2,T=10);
  Real x "State variable of continuous PI controller";
  equation
    der(x) = error/T;
    outCtr = K*(error+x);
  end PIcontinuousController;
  
```

base class for controllers – to be defined

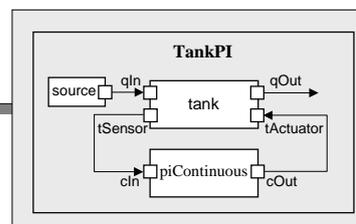
error – to be defined in controller base class

## The Base Controller – A Partial Model

```

partial model BaseController
  parameter Real Ts(unit="s")=0.1
    "Ts - Time period between discrete samples - discrete sampled";
  parameter Real K=2 "Gain";
  parameter Real T=10(unit="s") "Time constant - continuous";
  ReadSignal cIn "Input sensor level, connector";
  ActSignal cOut "Control to actuator, connector";
  parameter Real ref "Reference level";
  Real error "Deviation from reference level";
  Real outCtr "Output control signal";
  equation
    error = ref-cIn.val;
    cOut.act = outCtr;
  end BaseController;
  
```

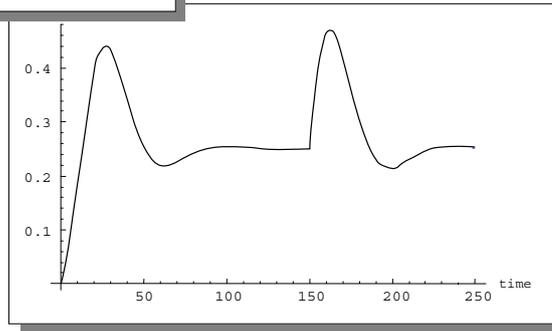
error = difference between reference level and actual tank level from cIn connector



## Simulate Component-Based Tank System

- As expected (same equations), TankPI gives the same result as the flat model FlatTank

```
simulate(TankPI, stopTime=250)  
plot(h, stopTime=250)
```

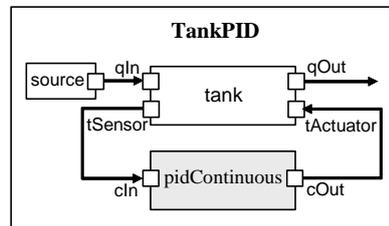


## Flexibility of Component-Based Models

- Exchange of components possible in a component-based model
- Example:  
Exchange the PI controller component for a PID controller component

## Tank System with Continuous PID Controller Instead of Continuous PI Controller

- Liquid source
- Continuous PID controller
- Tank



```

model TankPID
  LiquidSource      source (flowLevel=0.02);
  PIDcontinuousController pidContinuous (ref=0.25);
  Tank              tank (area=1);
equation
  connect(source.qOut, tank.qIn);
  connect(tank.tActuator, pidContinuous.cOut);
  connect(tank.tSensor, pidContinuous.cIn);
end TankPID;
  
```

## Continuous PID Controller

- error = (reference level – actual tank level)
- T is a time constant
- x, y are controller state variables
- K is a gain factor

$$\frac{dx}{dt} = \frac{error}{T}$$

$$y = T \frac{d \text{error}}{dt}$$

$$\text{outCtr} = K * (\text{error} + x + y)$$

*Integrating equations gives Proportional & Integrative & Derivative (PID)*

$$\text{outCtr} = K * \left( \text{error} + \int \frac{\text{error}}{T} dt + T \frac{d \text{error}}{dt} \right)$$

base class for controllers – to be defined

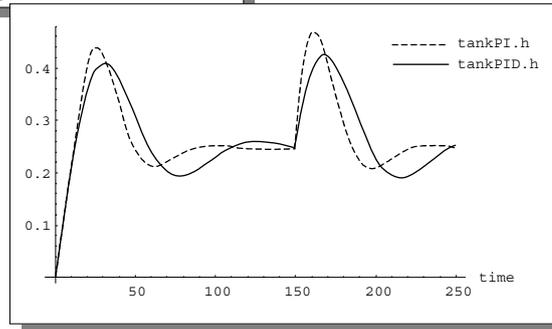
```

model PIDcontinuousController
  extends BaseController (K=2, T=10);
  Real x; // State variable of continuous PID controller
  Real y; // State variable of continuous PID controller
equation
  der(x) = error/T;
  y      = T*der(error);
  outCtr = K*(error + x + y);
end PIDcontinuousController;
  
```

## Simulate TankPID and TankPI Systems

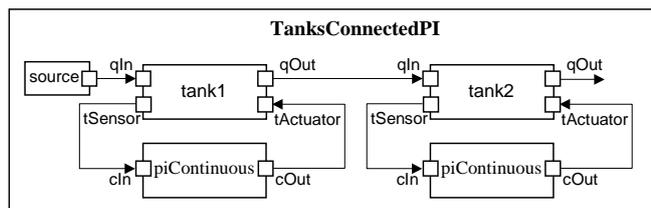
- TankPID with the PID controller gives a slightly different result compared to the TankPI model with the PI controller

```
simulate(compareControllers, stopTime=250)
plot({tankPI.h, tankPID.h})
```



## Two Tanks Connected Together

- Flexibility of component-based models allows connecting models together

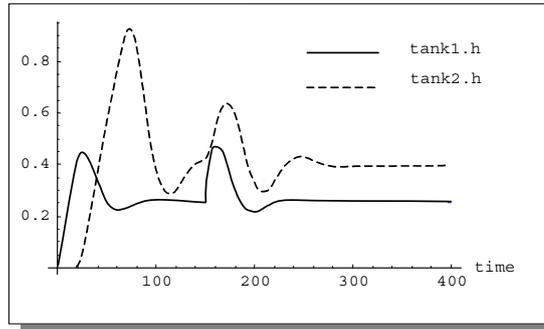


```
model TanksConnectedPI
  LiquidSource source(flowLevel=0.02);
  Tank tank1(area=1), tank2(area=1.3);
  PIcontinuousController piContinuous1(ref=0.25), piContinuous2(ref=0.4);
equation
  connect(source.qOut, tank1.qIn);
  connect(tank1.tActuator, piContinuous1.cOut);
  connect(tank1.tSensor, piContinuous1.cIn);
  connect(tank1.qOut, tank2.qIn);
  connect(tank2.tActuator, piContinuous2.cOut);
  connect(tank2.tSensor, piContinuous2.cIn);
end TanksConnectedPI;
```

## Simulating Two Connected Tank Systems

- Fluid level in tank2 increases after tank1 as it should
- Note: tank1 has reference level 0.25, and tank2 ref level 0.4

```
simulate(TanksConnectedPI, stopTime=400)
plot({tank1.h,tank2.h})
```



## Exchange: Either PI Continuous or PI Discrete Controller

```
partial model BaseController
  parameter Real Ts(unit = "s") = 0.1 "Time period between discrete samples";
  parameter Real K = 2 "Gain";
  parameter Real T(unit = "s") = 10 "Time constant";
  ReadSignal cIn "Input sensor level, connector";
  ActSignal cOut "Control to actuator, connector";
  parameter Real ref "Reference level";
  Real error "Deviation from reference level";
  Real outCtr "Output control signal";
equation
  error = ref - cIn.val;
  cOut.act = outCtr;
end BaseController;
```

```
model PIDContinuousController
  extends BaseController(K=2,T=10,Ts=0.1)
  Real x;
  Real y;
equation
  der(x) = error/T;
  y = T*der(error);
  outCtr = K*(error + x + y);
end PIDContinuousController;
```

```
model PIDiscreteController
  extends BaseController(K=2,T=10,Ts=0.1)
  discrete Real x;
equation
  when sample(0, Ts) then
    x = pre(x) + error * Ts / T;
    outCtr = K * (x+error);
  end when;
end PIDiscreteController;
```

## Exercises

- Replace the Picontinuous controller by the Pldiscrete controller and simulate. (see also the book, page 461)
- Create a tank system of 3 connected tanks and simulate.

## Principles for Designing Interfaces – i.e., Connector Classes

- Should be *easy* and *natural* to connect components
  - For interfaces to models of physical components it must be physically possible to connect those components
- Component interfaces to facilitate *reuse* of existing model components in class libraries
- Identify kind of interaction
  - If there is interaction between two *physical* components involving energy flow, a combination of one potential and one flow variable in the appropriate domain should be used for the connector class
  - If information or *signals* are exchanged between components, input/output signal variables should be used in the connector class
- Use composite connector classes if several variables are needed

## Simplification of Models

- When need to simplify models?
  - When parts of the model are too complex
  - Too time-consuming simulations
  - Numerical instabilities
  - Difficulties in interpreting results due to too many low-level model details
- Simplification approaches
  - Neglect small effects that are not important for the phenomena to be modeled
  - Aggregate state variables into fewer variables
  - Approximate subsystems with very slow dynamics with constants
  - Approximate subsystems with very fast dynamics with static relationships, i.e. not involving time derivatives of those rapidly changing state variables

# Exercises Using OpenModelica and MathModelica Lite

Version 2006-09-17

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## 1 Simple Textual Modelica Modeling Exercises

### 1.1 Try DrModelica with VanDerPol

Locate the VanDerPol model in DrModelica (link from Section 2.1), run it, change it slightly, and re-run it.

### 1.2 HelloWorld

Simulate and plot the following example with one differential equation and one initial condition. Do a slight change in the model, re-simulate and re-plot.

```
model HelloWorld "A simple equation"
  Real x(start=1);
equation
  der(x) = -x;
end HelloWorld;
```

### 1.3 BouncingBall

Locate the BouncingBall model in one of the hybrid modeling sections of DrModelica (e.g. Section 2.9), run it, change it slightly, and re-run it.

### 1.4 A Simple Equation System

Make a Modelica model that solves the following equation system with initial conditions:

$$\begin{aligned}\dot{x} &= 2 * x * y - 3 * x \\ \dot{y} &= 5 * y - 7 * x * y \\ x(0) &= 2 \\ y(0) &= 3\end{aligned}$$

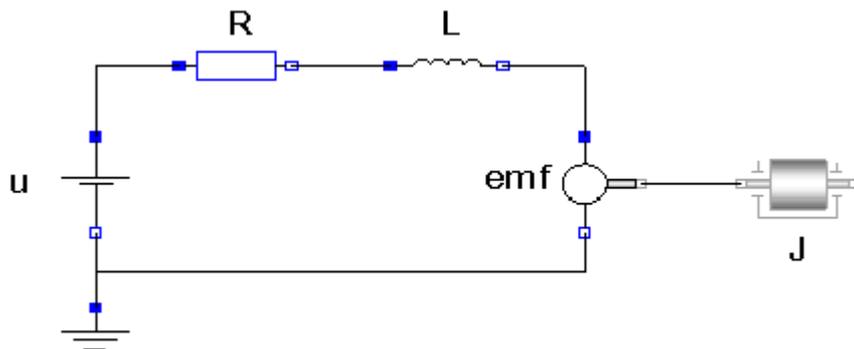
### 1.5 Functions and Algorithm Sections

- Write a function, `sum`, which calculates the sum of Real numbers, for a vector of arbitrary size.
- Write a function, `average`, which calculates the average of Real numbers, in a vector of arbitrary size. The function `average` should make use of a function call to `sum`.

## 2 Graphical Design using MathModelica Lite

### 2.1 Simple DC-Motor

Make a simple DC-motor using the Modelica standard library that has the following structure:

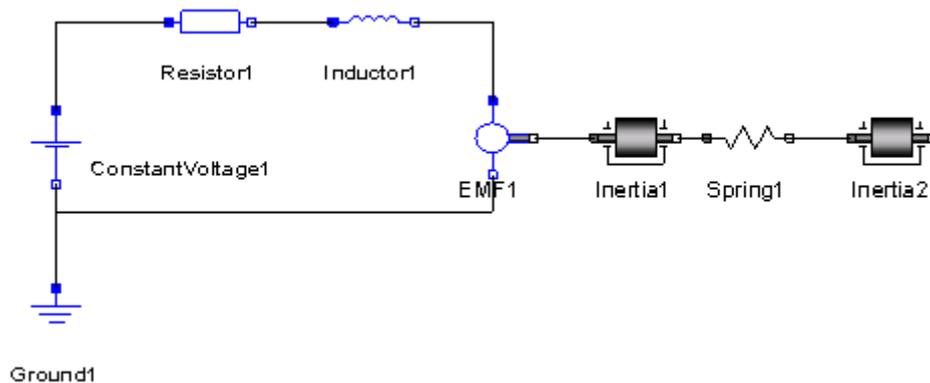


Save the model from the graphic editor, load it and simulate it (using OMSHELL or OMNotebook) for 15s and plot the variables for the outgoing rotational speed on the inertia axis and the voltage on the voltage source (denoted  $u$  in the figure) in the same plot.

Hint: if you have difficulty finding the names of the variables to plot, you can flatten the model by calling `instantiateModel`, which exposes all variable names.

### 2.2 DC-Motor with Spring and Inertia

Add a torsional spring to the outgoing shaft and another inertia element. Simulate again and see the results. Adjust some parameters to make a rather stiff spring.



### 2.3 DC-Motor with Controller (Extra)

Add a PI controller to the system and try to control the rotational speed of the outgoing shaft. Verify the result using a step signal for input. Tune the PI controller by changing its parameters in MathModelica Lite.

